1 (10 points) (a) An electron is located at \( \langle 3 \times 10^{-6}, 0, -2 \times 10^{-6} \rangle \text{m} \).
Sketch the electron and the field about it. What is the electric field \( \langle E_x, E_y, E_z \rangle \) at location at \( \langle 1 \times 10^{-6}, 0.4 \times 10^{-6} \rangle \text{m} \)?

\[ \begin{align*}
\vec{r} &= \langle 0.5 \rangle - \langle \text{source} \rangle = \left[ 1, 0, 4 \right] - \left[ 3, 0, -2 \right] \approx 10^{-6} \\
\hat{r} &= \left[ -2, 0, 6 \right] \times 10^{-6} \\
|\vec{r}| &= \sqrt{4 + 36} \times 10^{-6} = 6.3 \times 10^{-6} \\
\vec{E} &= \left[ -2, 0, 6 \right] \times 10^{-6} \\
&= \left[ -0.3, 0, 0.95 \right] \text{ m} \\
&= \frac{-0.3 \hat{j} + 0.95 \hat{k}}{6.3 \times 10^{-6} \\
&= 36 \frac{N}{C} \left[ -0.3, 0, 0.95 \right] \\
&= \left[ -10.9, 0, 34 \right] \text{ N/C}
\end{align*} \]

(b) An electron is now placed at location \( \langle 1 \times 10^{-6}, 0.4 \times 10^{-6} \rangle \text{m} \). In the space below draw an arrow representing the electric field you calculated, and label it. Calculate the magnitude of the force on the electron. Draw a second arrow representing the force on the particle, and label it.

\[ \begin{align*}
\vec{F} &= q \vec{E} = 1.6 \times 10^{-19} \frac{C}{\text{N/C}} \left[ -10.9, 0, 34 \right] \text{ N/C} = \left[ 17, 0, 65 \right] \times 10^{-19} \text{N} \\
&= \text{Directly away from the original } e^- \text{ along } \hat{r} \\
|\vec{F}| &= \sqrt{17^2 + 65^2} \times 10^{-19} \text{N} \\
&= 6.7 \times 10^{-19} \text{N}
\end{align*} \]
2. (10 pts) A metal sphere with radius 2 cm has a total charge of $-30 \text{nC} \left( -30 \times 10^{-9} \text{C} \right)$. It is surrounded by a hollow metal shell with inner radius 6 cm and outer radius 7 cm. The total charge on the hollow metal shell is $+50 \text{nC}$.

(a) At location B, draw a vector representing the electric field at that location, and calculate the magnitude of the electric field at B. Briefly explain your calculation, and include appropriate units.

(b) Show rigorously that there is $+30 \text{nC}$ on the inner surface of the hollow shell, and $+20 \text{nC}$ on the outer surface. Your explanation must be based on fundamental principles, not mere plausibility.

\[ |\vec{E}| = E_{pc} = \frac{1}{4\pi\epsilon_0} \frac{20 \times 10^{-9} \text{C}}{(0.25 \text{m})^2} = 2880 \text{N/C} \]

- Inner sphere has $-30 \text{nC}$ on outer surface b/c it is a conductor and $\vec{E} = 0 \text{N/C}$ inside.
- Inner surface of outer metal shell has $+30 \text{nC}$ b/c it is also a conductor and $\vec{E} = 0 \text{N/C}$ inside the outer shell.
- Outer shell outer surface has $g = -30 \text{nC}$ due to \( q \) above, plus a total charge imbalance of $+50 \text{nC}$ in addition.

At a distance this setup looks like a single point charge $g = +20 \text{nC}$.

10 points
3(a). (10pts) Using a sketched figure, describe the mechanism underlying the creation of a spark between one's finger and a doorknob on a dry day. Assume the hand has a net negative charge. Discuss why the spark starts and stops as it does, and where the light comes from. Discuss any assumptions or quantities.

- Initially neutral hand approaches initially neutral knob.
- When $|E| > 3 \times 10^6$ N/C, electrons in air accelerate enough to ionize $N_2$ and $O_2$. In air, freeing more $e^-$. restroom
- $e^- \rightarrow \oplus e^- \rightarrow @ etc$ Electron cascade when $|E| > 3 \times 10^6$ V/m creates spark.
- Spark rips through air between hand and knob. Flesh is due to recombining $N_2$ and $O_2$. Motion of atmospheric ions reduces $|E|$ between hand and knob, discharging hand and charging knob. Spark ceases when $|E|$ drops below $3 \times 10^6$ N/C, leaving knob negatively charged, hand somewhat discharged.

b. (5pts) A modern automobile spark plug is connected to a high voltage ignition system producing 40,000 Volts of electrical potential across a spark gap of 1.5mm. Calculate the maximum possible electric field strength between the electrodes of the spark plug. Discuss your number.

$$d = 1.5 \times 10^{-3} \text{ m}$$

$V = 40,000 \text{ V}$

$$E = \frac{V}{d} = \frac{40,000 \text{ V}}{1.5 \times 10^{-3} \text{ m}} = 27 \times 10^6 \frac{\text{V}}{\text{m}}$$

This is approx 10X the ordinary breakdown field strength of air ($3 \times 10^6$ N/C). 2p

The max field in the plug probably only gets to about $3 \times 10^6$ N/C, the extra field strength might only be needed under very adverse conditions in the fuel/air mixture. (Notice $e^- \cdots \text{e}^+$) 1/5
4. (15 pts) Two 10cm diameter inflated balloons of mass 2.5 g each are charged by rubbing them with fur, and suspended upon threads 0.65m long fastened together at the point of suspension. They repel one another so as to create a triangle with an angle of 60 degrees between the two threads at the point of suspension. (You have seen this apparatus in class.) Sketch and label this apparatus, and calculate the approximate charge on each balloon. Explicitly state and explain any simplifying assumptions.

\[ y = 0.70 \text{m} \times \sin 30^\circ = 0.35 \text{m} \]

\[ r = 2y = 0.70 \text{m} \]

Assumptions (2 pts)

- Replace balloons w/ 2 identical pt charges \( q \) and identical pt mass 2.5 g

\[
F_c = \frac{1}{4\pi \varepsilon_0} \frac{q^2}{r^2}
\]

\[
F_c = \frac{9 \times 10^9 \text{N} \cdot \text{m}^2 \cdot \text{C}^2}{(0.35 \text{m})^2} = 1.84 \times 10^4 \text{N} \cdot \text{C}^2
\]

\[
F_{grav} = m g = (2.5 \times 10^{-3} \text{kg})(9.8 \text{ N/kg}) = 0.025 \text{ N}
\]

\[
T_x = T \cos 60^\circ = F_c
\]

\[
T = \frac{F_c}{\cos 60^\circ}
\]

\[
T = \frac{F_{grav}}{\sin 60^\circ}
\]

\[
F_c = \frac{\cos 60^\circ}{\sin 60^\circ} F_{grav} \rightarrow 0.184 \times 10^4 \text{N} \cdot \text{C}^2 = \frac{\cos 60^\circ}{\sin 60^\circ} (0.025 \text{ N})
\]

\[
q = \sqrt{q^2} = \sqrt{\frac{\cos 60^\circ}{\sin 60^\circ} \cdot \frac{0.025 \text{N}}{1.84 \times 10^4 \text{N/C}^2}} = 8.9 \times 10^{-2} \text{ C}
\]

\[
q \approx 9 \mu \text{C} \text{ on each balloon.}
\]
5. (10 points) When operating an electrophorus, a neutrally-charged metal plate starts 10 cm away from a negatively-charged insulating styrofoam plate, then is moved to 1 cm away from the styrofoam plate and is touched with a finger drawing a static electric spark. Then the metal plate is returned to the styrofoam and touched again, drawing a spark a second time.

(a) Draw four diagrams which indicate how charge is distributed in each component of the electrophorus at each position (down, touch, up, touch). Explain this distribution.
(b) When is the metal plate charged by induction? By contact?
(c) In which direction does charge flow in each diagram? What evidence do you have of this?

1. As neutral foil approaches
   - positive styro, charge separation is
   - induced + e- rad to lip of foil

2. Spark jumps from foil to
   - finger, when finger touches
   - some excess e- is removed
   - from foil

3. Plate has electron deficit
   - or negative charge

4. Spark e-
   - Body is source of
   - static e-

   When hand is brought back
   near plate, it discharges tive
   plate by providing e- to restore
   plate to neutral charge

3. The plate is charged by induction when touched
   while held in E field of styrofoam. The plate is
   discharged by contact when touched in the air.

4. As shown. Can test signs of
   charge on plate w/ U & L tapes,

- can also track spark w/ a Neon bulb. Logic +
  observation of signs of charges we bulb.
6. (15 points) Three charges.

Three charges are fixed in a triangle as shown, with Q1 at the origin. Each is 5\(\mu\)C in magnitude, and 
\(R_{12} = R_{13} = 0.50\ \text{m}.

(a) Qualitatively describe the directions of the individual and total Coulomb forces exerted on Q1 by the other two charges. Notice symmetry makes this problem quite easy.

(b) What is the size of the Coulomb force between Q1 and Q2?

(c) What is the size and direction of the total Coulomb force exerted on Q1 by Q2 and Q3 together?

(d) If Q1 was removed from the origin, what is the size and direction of the total field at the origin due to Q2 and Q3?

\[ a\] By symmetry, \(\vec{F}_{Q2\text{on }Q1}\) and \(\vec{F}_{Q3\text{on }Q1}\) are the same size, and their y-components are equal and opposite so the \(\sum \vec{F}_y = 0\). Also, \(\sum \vec{F}_x = 2F_x(Q_{2\text{on }Q1}) = 2F_x(Q_{3\text{on }Q1})\). Very symmetrical.

\[ b\] 
\[
\sum \vec{F}_x = 2x \frac{1}{4\pi\epsilon_0} \frac{(5 \times 10^{-6})^2}{(0.50)^2} \cos 45^\circ = 1.3 \text{ N} \hat{x}
\]

\[ c\] 
\[
\vec{F}_{Q1\text{on }Q2} = \frac{1}{4\pi\epsilon_0} \frac{(5 \times 10^{-6})^5}{(0.50)^2} \text{ along } \hat{r} = -0.9 \text{ N along } \hat{r} \text{ (negative means attractive)}
\]

\[ d\] 
\[
\vec{E} = \frac{\vec{F}}{q} = \frac{1.3 \text{ N} \hat{x}}{5 \times 10^{-6} \text{ C}} = 2.6 \times 10^5 \text{ N/C} \hat{x}
\]
7. (10 pts). An Fe$^{3+}$ ion and a Cl$^{-}$ ion are positioned as shown below (all dimensions are in m).

(a, b) Calculate the strength and direction of the electric field at points A and B as labeled.
(c) If an electron is placed at A, what is the size and direction of the Coulomb force it experiences?

\[ 
\vec{E}_A = \vec{E}_{Fe^3+} + \vec{E}_{Cl^-} 
\]

\[ 
= \frac{+1 \times 1.6 \times 10^{-19} \text{C}}{4\pi \varepsilon_0 \left( \frac{3.0 \times 10^{-9} \text{m}}{100 \times 10^{-9} \text{m}} \right)^2} - \frac{1 \times 1.6 \times 10^{-19} \text{C}}{4\pi \varepsilon_0 \left( \frac{100 \times 10^{-9} \text{m}}{100 \times 10^{-9} \text{m}} \right)^2} \hat{x} 
\]

\[ 
= +0.48 \times 10^5 \text{N/C} + 1.4 \times 10^5 \text{N/C} \hat{x} \]

\[ 
\text{towards Cl}^{-} \text{ion} 
\]

\[ 
\vec{E}_B = \vec{E}_{Fe^3+} + \vec{E}_{Cl^-} 
\]

\[ 
= \frac{+1 \times 1.6 \times 10^{-19} \text{C}}{4\pi \varepsilon_0 \left( \frac{5.0 \times 10^{-9} \text{m}}{100 \times 10^{-9} \text{m}} \right)^2} - \frac{1 \times 1.6 \times 10^{-19} \text{C}}{4\pi \varepsilon_0 \left( \frac{100 \times 10^{-9} \text{m}}{100 \times 10^{-9} \text{m}} \right)^2} \hat{x} 
\]

\[ 
= 1.73 \times 10^4 \text{N/C} - 1.4 \times 10^5 \text{N/C} \hat{x} 
\]

\[ 
\text{towards Cl}^{-} \text{ion} 
\]

\[ 
F = q\vec{E} = (+1.6 \times 10^{-19} \text{C}) \left( +1.9 \times 10^5 \text{N/C} \hat{x} \right) 
\]

\[ 
= 3 \times 10^{-14} \text{N} \hat{x} \]

\[ 
\text{towards the Cl}^{-} \text{ion} 
\]