Find potential at \( A \) due to \( q_1 \) and \( q_2 \).

\[
V_A = \frac{1}{4\pi \varepsilon_0} \frac{q_1}{r_{1A}} + \frac{1}{4\pi \varepsilon_0} \frac{-q_2}{r_{2A}} - \frac{\text{sign}}{3}
\]

\[
V_A = \frac{1}{4\pi \varepsilon_0} \left( \frac{q_1}{r_{1A}} - \frac{q_2}{r_{2A}} \right)
\]

What is the potential at \( B \) due to ring of charge \( \phi \) or change?

\[
V_B = -\int \vec{E} \cdot d\vec{l} = -\int \vec{E}_{\text{ring}} \cdot d\vec{l}_{\text{distro}}
\]

\[
= -\int_{l_0}^{l_\infty} \vec{E} \cdot d\vec{l} = -\int_{l_0}^{l_\infty} \frac{\phi l}{4\pi \varepsilon_0 (l + \sqrt{l^2 + z^2})^{3/2}} dl
\]

\[
V_B = -\frac{\phi}{4\pi \varepsilon_0} \int_{l_0}^{l_\infty} \frac{l}{(l + \sqrt{l^2 + z^2})^{3/2}} dl
\]

\[
= +\frac{\phi}{4\pi \varepsilon_0} \left[ \frac{1}{(l + \sqrt{l^2 + z^2})^{1/2}} \right]_{l_0}^{l_\infty} = \frac{\phi}{4\pi \varepsilon_0} \frac{1}{\sqrt{(l + \sqrt{l^2 + z^2})}}
\]

\[
E_{\text{ring}} = \frac{\phi}{4\pi \varepsilon_0} \frac{l}{(l + \sqrt{l^2 + z^2})^{3/2}}
\]

Can also state by symmetry, all \( \phi \) is

\[
-\text{sign} \quad -\text{sign}
\]

- soln - argument/procedure

\[
V_B = V_{pc} = \frac{\phi}{4\pi \varepsilon_0} \frac{1}{r} = \frac{\phi}{4\pi \varepsilon_0} \frac{1}{\sqrt{l^2 + z^2}}
\]
RQ 16.8 Statements

For F + Why?

1. The electric potential inside a metal in a static equilibrium is always zero.” see p 558 § 16.47

**False** The change in electric potential inside a metal at static equilibrium is always zero. The potential can be defined arbitrarily to any value. \( V \) inside the metal at static eq = 0.

2. “If there is a constant large positive potential throughout a region, the electric field in that region is large.”

**False**. See figure of a high potential region, here the electric field self-cancel (imagine a +q0 at X) and is pretty small, despite a huge voltage. Big voltage with small \( \Delta V \), like on a gravitational plateau (large GPE with low local slope or no local slope).

3. “If you get close enough to a negative point charge, the potential becomes negative, no matter what other charges are around.”

**True**

\[
V = \frac{1}{4\pi\varepsilon_0} \frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0} \frac{q_2}{r_2} + \ldots
\]

One can imagine charges close together, \( q_1 = -e = -1.6 \times 10^{-19} \text{C} \), where \( q_2 = +100 \text{ e}^+ \)

\( q_1 = -e \quad q_2 = +100e \quad \Phi_2 \) will continue to create a positive electric potential near \( q_1 \) until we get to a distance \( r_1 \) from \( q_1 \) such that \( r_2 \) from \( q_2 = \frac{1}{100} r_1 \), where they will balance.

For any finite amount of charge, we can get infinitely close enough to a charge \( q_1 \) by it is a point charge. Hence this is theoretically true.
d) "Near a point charge, the potential difference between two points a distance L apart is \(-\vec{E}L\)"

\[=\ \text{False} \]

The L separation is not adequately defined. Two points on an equipotential can be L apart and have 0 potential difference, with nonzero E field strengths. (Along this, this also is not true, unless L is vanishingly small)

\[=\ "In a region where the electric field is varying, the potential difference between two points a distance L apart is \(-(E_f-E_i)\ L\)"

\[\text{False}\]

See argument above for d) L is undefined. Even when L lies along \(\hat{r}\) this is not true.

See p 565 "Avoiding a common pitfall"

"It is the electric field; the intervening region that defines \(\Delta V\)"

Dipole: both E and V varies
Problem 16.1 Potential along different paths in a capacitor

(a) For the path 1, A - B - C shown in the diagram, going from A to B we’re going directly opposite to the electric field (magnitude E) so the potential increases by an amount $E s_1$. From B to C the electric field is perpendicular to the path, so $E \cdot \vec{d}$ is zero. Therefore

$$\Delta V = V_C - V_A = E s_1$$

For path 2, A - C, we have

$$\int_{A}^{C} E \cdot \vec{d} = \int_{A}^{C} E_x dx = -(−E s_1) = E s_1$$

since all the $dx$’s are negative, but $E_x$ is positive (for x axis pointing to the right).

For path 3, A - D - B - C, along A - D there is no contribution (field perpendicular to path); along D - B the potential difference is $E s_1$ (same as A - C), and along B - C there is no contribution (field perpendicular to path). So again the potential difference is $E s_1$.

(b) $V_C - V_A = \frac{Q}{\varepsilon_0} s_1 = \frac{(+3 \times 10^{-6} \text{ C})}{\pi (4 \text{ m})^2} \frac{(0.7 \times 10^{-3} \text{ m})}{9 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}} = 67 \text{ volts}$

(c) For A - B - C - D, along C - D, $\Delta V$ is the opposite of A - B, so the round trip gives zero.

For A - B - A, again there is cancellation: A - B is $+E s_1$, and B - A is $-E s_1$, so the round trip gives zero.
16.2: Potential Difference along a wire

\[ \Delta V = V_B - V_A = 1.5 \text{ V} \quad \Rightarrow \quad \text{Since } 1.5 \text{ V} \text{ is a positive number, } V_B \text{ must be } 1.5 \text{ V higher in electric potential than } V_A. \]

\( \overrightarrow{E} \) goes from \( B \rightarrow A \); a test charge feels an electric force pushing it "downhill" in potential from \( B \rightarrow A \).

Note here \( \overrightarrow{E} \neq 0 \) inside a conductor. This nonequilibrium condition requires an outside agent expending energy to maintain, especially if there are any significant \( \# \) of \( e^- \) flowing through the wire. Here there is probably a battery connected to the wire.
$E_{\text{before}} = E_{\text{after}}$

$\Delta E_{3e} + kE_i = \Delta E_{3e} + kE_f$

$\Delta E_{3e} = \Delta kE$

$E Ed = \frac{1}{2} m v_f^2$

$\delta \Delta V = \frac{1}{2} m v_f^2$

$N_f = \sqrt{\frac{2 q \Delta V}{m_e}} = \sqrt{2 \left( \frac{1.6 \times 10^{-19} \text{C}}{9 \times 10^{-31} \text{kg}} \right) (15000 \text{ V})}$

$N_f = \sqrt{5.3 \times 10^{15}} = 7.3 \times 10^7 \text{ m/s}$ (pretty zippy! but less than $c = 3 \times 10^8 \text{ m/s}$)

check units: $\frac{I}{kS} = \frac{N \cdot m}{kS} = \frac{k}{S^2} \frac{m}{kS} \frac{m}{S^2} = \frac{m^2}{S^2} \sqrt{\frac{m^2}{S^2}} = \frac{m}{S} \sqrt{\frac{m^2}{S^2}}$

electrons emerge at about 24% of the speed of light!
Problem 16.14 Electron deflection in an oscilloscope

The electron leaves the filament essentially at rest. It is accelerated to the right through a potential difference $\Delta V_{acc}$ and emerges with momentum $p_x$ in the $x$ direction. Next it passes through two horizontal deflection plates. During the time it is between these plates, it experiences a force in the $+y$ direction due to the electric field between the plates. It emerges with horizontal components $p_x$ (unchanged) and $p_y$. Assuming that the fringe fields of both the accelerating plates and the deflection plates are negligibly small, the electron then travels in a straight line until it hits the screen.

1) change in $p_x$ (initially 0):
$$\Delta K_{electron} = -\Delta U_{electron}$$
$$\Delta \left( \frac{p_x^2}{2m} \right) = \left( e \Delta V_{acc} \right) \quad (\Delta U < 0, \text{ so } -\Delta U > 0)$$
$$p_x = \sqrt{2em\Delta V_{acc}} = mv_x$$

2) Time to travel through deflection plates:
$$t_{plates} = \frac{L}{v_x} = \frac{mL}{p_x}$$

3) $E$ inside deflection plates:
$$\Delta V_{def} = -E_{def} \cdot Y \Rightarrow |E_{def}| = \frac{\Delta V_{def}}{Y}$$

4) change in $p_y$ (initially 0):
$$p_y = \Delta p_y = F_y \Delta t = eE_{def}t_{plates} = eE_{def} \left( \frac{mL}{p_x} \right)$$

5) To simplify the problem, assume that $\Delta y$, the $y$ distance traveled while inside the deflection plates, is negligibly small compared to the $y$ distance traveled after leaving deflection plates. The angle to the horizontal after leaving the plates is given by
$$\tan \theta = \frac{p_x}{p_x} = \frac{Y}{0.3 \text{ m}}$$

Now evaluate this ratio in terms of the other given quantities:
\[
\frac{y}{0.3 \ m} = \frac{P_y}{P_0} \ \\
\left( eE_{def} \frac{mL}{P_x} \right) \left( e^{-\frac{eV_{def}}{s}} \right) = \frac{\Delta V_{def}}{\Delta V_{acc}} \left( \frac{L}{s} \right) \\
= \frac{P_y}{P_0^2} \left( \frac{\Delta V_{def}}{2 em\Delta V_{acc}} \right) \left( \frac{emL}{s} \right) = \frac{1}{2} \left( \frac{\Delta V_{def}}{\Delta V_{acc}} \right) \left( \frac{L}{s} \right) \\
= \frac{1}{2} \left( \frac{40V}{(18000V)} \right) \left( \frac{L}{s} \right) \\
y = (0.3 \ m) \left( \frac{40V}{2 \ (18000V)} \right) \left( \frac{8 \times 10^{-2} m}{3 \times 10^{-3} m} \right) = 8.8 \times 10^{-3} \ m
\]

It was a good idea to wait until the end to put in numbers, since most of the quantities dropped out, and we avoided lots of unnecessary calculations!

Let's make sure that the electron misses the top deflection plate, and that \( \Delta y \), the \( y \) distance traveled inside the deflection plates, is in fact negligible compared to \( y \approx 1 \ cm \), as we assumed.

\[
v_x = \frac{2e\Delta V_{acc}}{m} = \sqrt{\frac{2(1.6 \times 10^{-19} C)(1.8 \times 10^4 \text{volts})}{(9 \times 10^{-31} \text{kg})}} = 8 \times 10^7 \ m/s
\]

\[
L_{planes} = \frac{L}{v_x} = \frac{8 \times 10^{-2} m}{8 \times 10^7 \ m/s} = 10^{-9} \ s
\]

\[
\Delta y = \frac{1}{2} a_{planes} t_{planes}^2 = \frac{1}{2} \frac{e}{m} \left( \frac{\Delta V_{def}}{\Delta V_{acc}} \right) \left( \frac{L}{s} \right) \left( \frac{40\text{volts}}{2(9 \times 10^{-31} \text{kg})(3 \times 10^{-3} m)} \right) (10^{-9} \ s)^2 = 1.2 \times 10^{-3} \ m
\]

which is less than \( \frac{s}{2} = 1.5 \times 10^{-3} m \)

So the electron does not hit the top plate, and \( \Delta y << y \).
\[ \vec{C} \text{ lies along } x \text{-axis: } \vec{C} = \langle 3, 0, 0 \rangle \]
\[ \vec{D} \text{ lies in } x-y \text{-plane: } \vec{D} = \langle 5 \cos 30^\circ, 5 \sin 30^\circ, 0 \rangle \]

Find \( \vec{C} \times \vec{D} \) and \( \vec{D} \times \vec{C} \).

\[
\vec{C} \times \vec{D} = \begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
3 & 0 & 0 \\
5 \cos 30^\circ & 5 \sin 30^\circ & 0
\end{vmatrix} = (0 \times 0 - 2.5 \times 0) \hat{i} + (0 \times 4.3 - 3 \times 0) \hat{j} + (3 \times 0 - 4.3 \times 0) \hat{k}
\]

= \(0 \hat{i} + 0 \hat{j} + 7.5 \hat{k}\)

\[ \alpha = \langle 0, 0, 7.5 \rangle \\
3 \text{ pts} \]

\[ \vec{D} \times \vec{C} = \begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
4.3 & 2.5 & 0 \\
3 & 0 & 0
\end{vmatrix} = (2.5 \times 0 - 0 \times 0) \hat{i} + (0 \times 3 - 4.3 \times 0) \hat{j} + (4.3 \times 0 - 3 \times 2.5) \hat{k}
\]

= \(0 \hat{i} + 0 \hat{j} - 7.5 \hat{k}\)

\[ \alpha = \langle 0, 0, -7.5 \rangle \\
3 \text{ pts} \]

Check: Rotating \( \vec{C} \) into \( \vec{D} \) goes along \( \hat{z} \) by RHR!!

Check: Rotating \( \vec{D} \) into \( \vec{C} \) goes along \( -\hat{z} \) by RHR

Note: \( \vec{C} \times \vec{D} = -\vec{D} \times \vec{C} \)

in vector cross multiplication, the order matters!!

3 pts / 16