UNIT V--CDP MODEL EXPLAINED

INSTRUCTIONAL GOALS

1. Newton's 2nd law
   Develop mathematical models from graphs of acceleration vs force and acceleration vs mass
   Introduce mathematical notation for a proportion
   introduce joint variation
   \[
   a \propto F_{net}, \quad a \propto \frac{1}{m} \quad \Rightarrow \quad a \propto \frac{F_{net}}{m}
   \]
   determine overall proportionality constant
   \[
   a = k \frac{F_{net}}{m} \quad \Rightarrow \quad k = \frac{ma}{F_{net}}
   \]
   show that \( k = 1 \) because of the way the newton was defined
   \[
   a_s = 9.8 \frac{m}{s^2} \quad \Rightarrow \quad g = 9.8 \frac{N}{kg}
   \]

2. CDP’s dynamical properties, force diagrams and motion maps
   2nd half of Newton's modeling cycle: from forces deduce motions
   Relate the directions of the acceleration and net force vectors
   resolve vectors into components

3. Friction
   Develop frictional force law
   Distinguish between static and kinetic friction
   \[
   F_k = \mu_s F_g
   \]
   \[
   F_s \leq \mu_s F_g
   \]
LAB NOTES

APPARATUS -- Modified Atwood's Machine Lab
wheeled carts             dynamics carts             glider
wood ramps               PASCO tracks             airtracks
pulleys with clamps      balance for mass measurement
hangers for slotted weights (or equivalent)
spring scales (newton calibration)
photogates               ULI Timer or MacMotion software
Graphical Analysis

PRE-LAB DISCUSSION -- Modified Atwood's Machine Lab
• Allow a suspended mass to tow a cart (glider) across the track; ask students to observe its motion. We've already established that a force is required to produce an acceleration. We just haven't quantified the relationship. Rather than brainstorming general observations, ask them to identify other factors that might affect the acceleration of the cart. To proceed, the list must include mass, amount of friction, and amount of force used to tow cart.
• Ask them for ideas on how to minimize the effect of friction. After some discussion, they will hopefully come to the idea of inclining the ramp slightly to compensate for friction.
• Ask them how to measure the acceleration of the cart. While they cannot measure it directly, there are at least two ways to do determine the acceleration. One can calculate it from rearrangement of the kinematical model \( \Delta x = \frac{1}{2} at^2 \). (Note: The use of this model requires the assumption that acceleration is constant. The rationale for such an assumption could be based on an "extra credit" lab.) Another method is to allow a picket fence affixed to the cart to pass through a photogate. The slope of the velocity vs time graph yields the acceleration.
• The dependent variable is the acceleration of the cart.
• The independent variables are the mass of the cart/hanger system and the force used to pull the cart.
• Make sure to stress that the mass that is being accelerated is the total mass of the system (the cart and hanging mass are connected, so must accelerate at the same rate).

LAB PERFORMANCE NOTES -- Modified Atwood's Machine Lab
• Use small mass hangers (e.g. 5g) and change by 10 g increments.
• Increase cart mass by 0.2 - 0.5 kg increments.
• Adjust the angle of incline so that the cart can move at a constant speed with a very small initial push.
• Convince students that they must transfer mass from the cart to the hanger in order to keep the total mass constant when they vary the force.
• Convert the hanging mass to newtons.
• See sample graphs in Figures 1, 2, and 3.
POST-LAB DISCUSSION -- Modified Atwood's Machine Lab

- Since units of slope are not intuitive, focus on proportionalities.
- Discuss the combination of two proportionalities into one:

  \[ a \propto \frac{F_{\text{net}}}{m} \quad a \propto \frac{1}{m} \quad \Rightarrow \quad a \propto \frac{F_{\text{net}}/m}{m} \]

- Turn proportionality into equation, rearrange to solve for \( k \).

  \[ a = k \frac{F_{\text{net}}}{m} \Rightarrow k = \frac{ma}{F_{\text{net}}} \]

- Substitute values from regression line to solve for \( k \). With luck, students' values should cluster around 1.0. Now is the time to point out that the slope of force of gravity vs mass (9.8 N/kg) and the slope of velocity vs time (9.8 m/s²) have the same numerical value due to the way the newton was defined.

FRICION

At this point you must decide to what depth you wish to treat friction. If you wish to perform a demonstration, you can use a force probe and motion detector with a "sled" and various masses. Load MacMotion software and set up the motion detector. Calibrate the force probe, then re-zero it in the horizontal position. If you steadily increase the pull on the block, the force vs time graph should show a steady increase until the applied force exceeds the force of static friction and the object moves at constant velocity (use motion detector to check this). One can obtain the coefficients of static and kinetic friction from the maximum and steady state values of applied force and the normal force.

If you wish a more thorough treatment, read the following

Pre-Laboratory Session

- Students are asked to make observations of an object being dragged along a surface. All observations are accepted. Friction is likely to be among the observations.

- Students are asked what affects the frictional force on the object and the surface. Ask what can be measured or otherwise described. Things like speed, weight, area of contact, and types of surfaces are important for students to mention.
• Guide students to the notion that it is the “support” force (normal force), rather than the force of gravity on the object, that really is the significant thing. Showing a situation in which the two are noticeably different can lead students to articulate this conclusion.

• The possible difference between static and kinetic friction may be elicited by using a rather massive box on the desk. The static frictional force may be seen to be a variable dependent on the pulling force applied by the student, on surface area, on surface type, and on support force. Therefore, static friction can be included in this experiment as desired. It may be desired to focus student attention on the maximum static friction force as the significant variable which can be graphed against the independent variables other than pulling force, because it is the single unique value that we can associate with static friction.

• Materials available are shown to students. Force scales, objects and surfaces, mass sets, paper, plastic, and sandpaper (to change surface types), a motor to drag objects along at various speeds, and a photogate system to time the object as it passes, and a meter stick should be among the apparatus available. Perhaps small pieces of wood of varying area having sandpaper, plastic, and paper attached can ride along with the object and serve as means to change surface area.

• If felt is one of the materials used for the surface type variable, some dependence on surface area may be observed. It appears that the increased mashing of the felt when a smaller surface area is used produces a change in the surface character. Therefore, felt may be used as desired. It depends on what the instructor wants the students to think about.

Laboratory Session

• A central part of this experiment will be the control of variables. Naturally, no help should be given students regarding this other than asking them to explain what they are doing and why and to refer lost students to others who are not so lost. The frictional force will be plotted against support force, surface area, speed, and type of surface, so there are several variables.

• The frictional force vs surface type graph is of a different kind than previous graphs, since surface type is not a continuous variable; a bar graph will be needed, but expect all kinds of graphs to be made. Through discussion about the meanings of incorrectly constructed graphs, the significance of bar graphs can become clear to students. Let them make the graphs they wish, but discuss them carefully in the post-laboratory discussion. It can be helpful during the laboratory session to find students making inappropriate graphs to present their results later.

POST-LAB DISCUSSION -- Friction Lab

• Develop the concepts that the friction force: (a) is independent of the contact surface area; (b) depends on the normal force (not always weight) and is different for static versus kinetic situations; and (c) depends on the types of surfaces in contact.

![Graphs showing force vs weight, surface area, and velocity](image-url)
• There may be some dependence of frictional force on speed. It should be small, however. Its existence is enough to note.
• A fairly linear relationship should be apparent from the force of friction vs support force graph. The coefficient of kinetic friction is introduced here.
• No mathematical model will be forthcoming from the graphical model of frictional force vs surface type. It should be concluded that each pair of surfaces will have its peculiar coefficient of friction.
• Aim for student application of the formulas:
  \[ F_s \leq \mu_s F_N \]
  \[ F_k \leq \mu_k F_N \]

**INSTRUCTIONAL COMMENTS**

In this unit, students learn the second half of Newton's Modeling Cycle:
a) from motions (read: changes in velocity) infer forces
b) from force deduce motions

Students should be able to correctly describe the kinematic behavior of an object from the force diagram. Typically, students are expected to be able to determine the net force, then value of unknown applied force from a description of the object's kinematic behavior.

In the deployment worksheets reinforce the practice of drawing force diagrams as the first step in preparing a solution to the problem. Make sure that these diagrams faithfully represent the forces (long-range and contact) that act on the object. In whiteboard presentations try to induce students to recognize multiple approaches to problems dealing with systems that consist of more than one object.