Review of Activities for Teaching the Inverse Square Law in the NYSED Regents Physics Classroom

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Abstract

It is an important part of learning that a person sees and engages a concept several times before mastery is attained (Arons, 1997). One such concept is the inverse square law. Many concepts in the NYS physics curricula can be explained by the inverse square law. Here I present a collection of activities that can be used throughout the school year to allow students to revisit the inverse square law using Newton's law of universal gravitation, Coulomb's law, and apparent brightness. The activities presented are connected to their associated New York State Regents Standards.

 Here I present a compilation of inverse square law activities that allow students to experimentally prove the relationship to themselves for various physical systems. These activities allow students to visualize the effects of the inverse square law in areas such as gravitational forces, electrostatic forces, and the apparent brightness of light sources. These activities have been chosen to provide “non-threatening” methods for teaching the inverse square law in a high school physics classroom, but they can easily be adapted for upper or lower level classes as needed.

Problem

 The inverse square law explains many physical phenomenon. The inverse square law's discovery is rooted in Isaac Newton's equation for universal gravitation, but it has been extended to other situations including Coulomb's Law, radiation, the description of apparent brightness of light waves, and Gauss' Law. The inverse square law is a geometric law involving spheres and therefore it is limited to situations that are associated with point-source symmetry.

*Insert Figure 1 here*

 The inverse square law is a common concept presented on final examinations in physics. A question from a New York State Regents Physics examination (June 2009, p. 3, question 10) looked like this (<http://www.nysedregents.org/physics/20090624exam.pdf>):

*A distance of 1.0 meter separates the centers of two small charged spheres. The spheres exert gravitational force Fg and electrostatic force Fe on each other. If the distance between the spheres’ centers is increased to 3.0 meters, the gravitational force and electrostatic force, respectively, may be represented as*

Responses: $\left(1\right) \frac{F\_{g}}{9} and \frac{F\_{e}}{9} \left(3\right) 3F\_{g} and 3F\_{e}$

 $\left(2\right) \frac{F\_{g}}{3} and \frac{F\_{e}}{3} \left(4\right) 9F\_{g} and 9 F\_{e}$

When students look at this problem there are many things they could misunderstand when reading it, but even if they do interpret the problem correctly, there is a misconception about scaling that causes many students problems. According to Arons (1997), “the great majority of students, including those in engineering-physics courses, have very serious difficulty with such questions. . . . they have no idea what to do in the absence of formulas for the relevant areas and volumes.” (p. 12)

 The problem students have when studying equations related to Arons' statement is that students tend to memorize those equations without understanding what each means and how those equations affect their results. And since the students lack understanding, they have difficulty using this information for any problems other than the ones explicitly presented to them in their classrooms and books.

It is illusory to expect to remediate these difficulties with a few quick exercises, in artificial context, at the start of a course. Most students can be helped to close the gaps, but this requires repeated exercises that are spread out over time and are integrated with the subject matter of the course itself. (Arons, 1997, p. 1)

As a result, it is important that the inverse square law be visited and revisited throughout a student's academic career and in various forms in order to expect them to understand it as well as is necessary for mastery.

Inverse Square Activities

*Insert Table 1 here*

*Introductory Activities*

 The simplest demonstrations of the inverse square law involve area measurements from a point-source of light. Two activities that use this method are *Data Analysis for the Inverse Square Law*, (Downie, 2007) and *Rainbow Glasses and the Inverse-square Law* (Doebler and George 1994). Additionally, the ASU Modeling Curriculum (Hestenes, 2006) also introduces the inverse square law as a part of its unit on Scientific Thinking, *Unit 1 - Worksheet 1: Graphing Practice* (modeling.asu.edu/Curriculum.html).

 Downie's (2007) activity for the inverse square law involves the use of a slide projector or other light source with the ability to project an image. The image in this activity is a series of uniform dots aligned in a grid formation. The students measured how far the projector was from a screen with an area marked on it. While the projector was in this position, the students counted the number of dots that are in the marked area. Students repeated this process for various distances between the projector and the screen. Once the data was collected, the students plotted the data on a graph with axis of number of dots versus distance from the screen and came up with an inverse square relationship on their graphs. Students may not immediately know how to determine what the equation for this curve is, but that is dependent on the instructor's expectations, guidance, and the level of the students.

 The activity presented by Doebler and George (1994) requires rainbow glasses, sometimes known as fireworks glasses. A company called Rainbow Symphony builds glasses made from a two-dimensional diffraction grating that causes light traveling through them to split into beams of light that form a grid. This grid forms a nearly perfect set of points to form a square on the screen in front of the glasses when a monochromatic laser is shone through them. The students assembled the laser and glasses setup and set it at various distances from a screen. The students measured the distance to the screen and determined the area of the square projected on the screen from each distance. The students then plotted the graph of area versus the square of the distance from the screen. This results with the students getting a graph they could draw a straight line of best-fit for and determine an equation.

 The ASU modeling activity, *Unit 1 – Worksheet 1: Graphing Practice*, is a graphical representation activity where students explore various data sets and use data analysis tools to determine the equations for the functions. After a brief introduction using notes and teacher guidance the students plotted graphs and visually analyzed the data to determine what action should be done to the graph axis to turn it into a straight line graph. Some of the data sets in this activity allow students to see what a data set looks like for an inverse square relationship (Hestenes, 2006).

 Each of these activities have their own strengths and weaknesses when being applied to the classroom. It is up to the instructor to determine which activity is best for their students' needs. The Rainbow Glasses activity makes mention of the cost effectiveness of the glasses and lasers, but needs to be explicitly linked to the inverse square law. This link needs to be made because the data associated with the rainbow glasses activity implies that the distance from screen is proportional to the area of the square. This is not the inverse relationship the students are attempting to prove. Therefore the teacher must make the explicit connection that the area of the square is inversely proportional to the brightness of the screen at each distance. Downie’s Dots and Projector activity provides a direct data set for the inverse square law but the activity requires the use of a projector for each group. The Graphing Practice activity is fast and efficient but provides no data collection activity which many students may need to effectively interpret the concept properly.

*Gravitational Activities*

 Another place in the physics curriculum to reinforce the inverse square law is in the section for gravitational force and motion. This is a difficult topic to show students the connection because we simply cannot vary the gravitational force for easy experimentation in a classroom setting. But some teachers have made it possible for students to make measurements in their classrooms based on games and geometric activities.

 One activity that creates a strong conceptual example of the gravitational relationship to the inverse square law is *Teaching Universal Gravitation with Vector Games* (Lowry, 2008). In this activity, Lowry takes the “Space Race” concept presented by Vinson (1998) and adds his own twist by creating a black hole on the game board. In addition to the normal moves originally in the game, Lowry's “Space Race” incorporates the black hole by having students measured their “ship's” distance from the black hole. The students then squared that distance and used the information to apply the black hole effects to their ship. Lowry states, *that many of his students are surprised by the drastic differences in motion caused by the black hole and that they gain a strong conceptual understanding of the gravitational effects in relation to the inverse square law which can then be referred to during explicit instruction* (Lowry, 2008, p. 521).

 A second activity that can be used to introduce the inverse square law in relation to gravitational effects was presented by Prentis, Fulton, Hesse, and Mazzino (2007). Their Elliptical Orbit activity goal was to illustrate a geometric method to prove that the force of gravity on celestial objects is actually an inverse square relationship. In the activity, students laid out the typical orbit activity with two pins as the foci of the elliptical orbit and a string to mark the orbit of the “planet” around its star. The students then used Newton's assertion that for infinitesimal sections of an elliptical orbit, the path of the object is nearly parabolic. The students then measured the deviation from the tangent of the object in its orbit. This deviation was compared to the time interval of that portion of the orbit. The time interval is based on the area of the orbit being swept out during that interval. After completing all of the measurements and calculations associated with the Elliptical Orbit activity, the students came to a result that showed that the gravitational force has an inverse square relationship.

 Each of these activities has its mathematical and conceptual strengths. Both activities require students to use equations that involve the inverse square component to be successful and allow students to explore the deviation from normal everyday experiences. However, both of them depart from realistic situations in order to accomplish their goals of teaching the inverse square law's effect on gravity. The Space Race activity this accomplishes the task of effectively teaching the students by making it fun and enjoyable to learn. The Elliptical Orbit activity uses an extreme situation of a strongly elliptical orbit to generate data. But both seem to be useful in their arenas and are very effective for keeping the mathematical portion of the activity low while producing results the students can understand. Additionally, a website produced by the University of Colorado at Boulder (phet.colorado.edu) has many web applets that show promise in demonstrating gravitational motion and effects based on the inverse square law.

*Coulomb's Law Activities*

 Two methods of introducing Coulomb's Law in the classroom were presented by Cortel (1999) and another by Mahoney (1971). Although both of these activities are a little older, they both present Coulomb's Law in a simple manner that also allows the teacher to present the inverse square law to their students. Another brief mathematical activity based on Coulomb's Law that also demonstrates the inverse square law can be found in the ASU Modeling Curriculum (Hestenes, 2006). The ASU activity is from the E&M semester materials, *UNIT 1 – Mapping the Electric Field* (modeling.asu.edu/Curriculum.html).

 In Cortel's (1999) activity, the students used an electronic balance and two metallic spheres to show that the electrostatic force is inversely proportional to the square of the distance between their centers and it also allows those students to see the proportional relationship of force and charge. To focus on the inverse square nature of the activity, students set up a stationary platform with one of the metallic spheres attached to it on the balance. Students then set up the other sphere on a moveable ring stand hanging over the first sphere. The spheres were both charged with like charges and the students measured the force applied to the scale at various distances between the spheres. The students then plotted the data in the form of force versus distance. Using data analysis tools, students can easily determine that the force is related to the inverse square of the distance between the objects.

 Alternatively, Mahoney's (1971) activity is a class demonstration activity created for teaching students using an overhead projector. He set up a grid on an overhead projector with one conductive ball hanging from the top of the projector from some nylon strands and the other conductive ball was controlled by being attached to a wooden block. Both spheres were charged the same and the entire class could see the spheres reactions and motions on the wall of the classroom. Everyone could take the measurements from the class example. The students used their observations to measure two things: 1) the distance between the two balls on the screen, and 2) the displacement of the hanging sphere from its equilibrium position. The displacement of the hanging sphere was used as a substitution measurement for the force between the spheres. So the students in these classes graphed the relationship between the distance between the balls and the displacement “force” to discover that the relationship between them is the inverse square law.

 *Mapping the Electric Field* is a brief activity that tells students to calculate the force on a test charge at various positions surrounding a central charged object using Coulomb's Law. When the students calculated the forces, the students then drew vectors to represent the forces at those locations. Students are often surprised to find out how much stronger the force is when they get close to the central charge. However, in this situation, the students are not discovering the inverse square law and are only discovering the effects of it. This can still be a very useful activity for them. (Hestenes, 2006)

 It is important to realize that with both the Electronic Balance activity and the Overhead demonstration that continuous charge leak will cause error in measurements if not properly prepared for. As we had with the area demonstrations of the inverse square law, these Coulomb's Law verification activities provide teachers with both costly and inexpensive methods for teaching students the inverse square law with varying levels of accuracy. In this particular grouping, it is interesting to note that the more expensive method requires fewer intellectual leaps to grasp the understanding as the measurements are already in the proper units. Though electronic balances are becoming less expensive, the degree of accuracy necessary may be a limiting factor in deciding which balances you require for your classroom to complete the Electronic Balance activity. The overhead demonstration may take a little more time in class to explain that the distance moved by the dependent object is going to be proportional to the force between the objects. Depending on the level of the students, a proof of the relationship might also be necessary. The Mapping the Electric Field activity is not a very hands-on activity, but does get the point across in a fast and efficient manner.

*Wave-front Intensity*

 One last major area of introductory physics that the inverse square law can be emphasized is in the study of waves. Two activities presented here use light sources to demonstrate that the apparent intensity of the light from a point-source also follows the inverse square law. One of those activities was presented by Bohacek and Gobel (2011) and the other by Narayanan and Narayanan (1999).

 In the Laptop Screen activity by Bohacek and Gobel (2011), students used a laptop screen to simulate a point-source of light and measure the intensity of light hitting a light sensor at various distances from the computer screen. They then plotted the data of intensity versus distance and determined that this is another inverse square relationship. But these two authors took the activity further to show how the point-source situation is different from line-source or planar-source relationships. This may be a helpful connection to make if the students have had the inverse square law drilled into them for a strong part of the year or just to allow students to visualize examples of different fields that can be linked with the students' study of electric fields.

 The Airy's Disk activity produced by Narayanan and Narayanan (1999) also makes use of a photometer to measure the strength of the light at various distances from its source. They accomplish this by shining laser light through a pinhole to produce Airy's disk. The amount of illumination received by the photometer at various distances is then graphed in relation to the distance from the screen. An inverse square relationship is clearly presented. This activity calls for a photometer that can be set to detect a specific wavelength of light.

 These wave-front intensity activities are easily connected back to the area measurements made at the beginning of this article. Both require a little more technology than some of the previous sections due to the nature of light and the technology necessary to make these measurements. But these activities also provide data that is in the proper units and that fits nicely into the equation for the inverse square law and apparent brightness. There are no leaps of intuition or difficult calculations to make. The Airy's Disk activity does seem to require a little more explanation and more expensive equipment to complete, due to the nature of Airy's disk and the need for a higher precision photometer.

New York State Curriculum

 Due to the exploratory nature of these lessons, they can easily be linked with whatever level of science that is being taught. The major connections will be explicitly linked below, but for a full list of related NYS Regents Physics standards see Table 2.

*Insert Table 2 here*

 Since the format of almost all of these lessons involve a laboratory exercise and graphing of some sort, these activities share many common NYS standards. One of the most notable is Standard 1 – Mathematical Analysis Key Idea M1.1. This standard states that students should “use algebraic and geometric representations to describe and compare data.” This standard is shared by every activity presented and provides a framework for allowing students to develop their own models of the observed phenomena. The other standard that quite obviously links all of these activities is Standard 4 – The Physical Setting Performance Indicator 5.1u which states that students should understand that “the inverse square law applies to electrical and gravitational fields produced by point sources.” This is a clear indication of the importance of this concept in the NYS curriculum (University of the State of New York State Education Department, 2011).

 Additionally, many other standards are shared between these activities but a few stand out as being more strongly shared. Two of them are further associations with Standard 1 – Mathematical Analysis, Key Ideas M2.1 and M3.1. A third strongly represented connection is Standard 6 – Patterns of Change Key Idea 5.2. These three standards are shared by more than 75% of the suggested activities and further represent the commonality of graphical and algebraic representations as well as the recognition of patterns that develop in the collection of data (University of the State of New York State Education Department, 2011).

Conclusion

 These activities were chosen based on my experiences within the classroom and what I would like to try to use for my classes in the future. I have only previously tried the two activities associated with the ASU Modeling Curriculum and I feel that they have both strengthened my students' understanding of the inverse square law. I feel that investigations of the other activities presented here will greatly increase a student's understanding of the inverse square law.

 Most people cannot learn a concept like the inverse square law without repeatedly visiting the content and practicing the material. The activities presented here are designed to help students along the path to understanding and assist in retaining the concepts after instruction. They provide students with different representations and various situations to realize the connection and common factors between these phenomena. The use of these activities is of course at the teacher's discretion, and you may find that certain activities benefit your classroom better than others. But be sure to revisit the concept and make those connections for your students to be successful.

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About the Author

 I graduated from Buffalo State College in 2006 with a Bachelors of Science and Education for grades 7-12. I also completed the Masters of Science in Education program for Physics at Buffalo State College in 2011. Since 2006 I've taught 4 years of high school Physics and Environmental Science in Baltimore City, MD Public Schools and Buffalo, NY Public Schools. I also spent one year in Japan teaching English in public schools from 2011 to 2012.

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Table 1

A Summary of Inverse Square Law Activities and Materials

Table 2

Correlations Between New York State Core Physics Curriculum and Reviewed Activities

Table 3

Strengths and Weaknesses of the Reviewed Activities

Image 1

Spherical Scaling

