A Review of Activities for Teaching the Inverse Square Law in the NYSED Regents Physics Classroom

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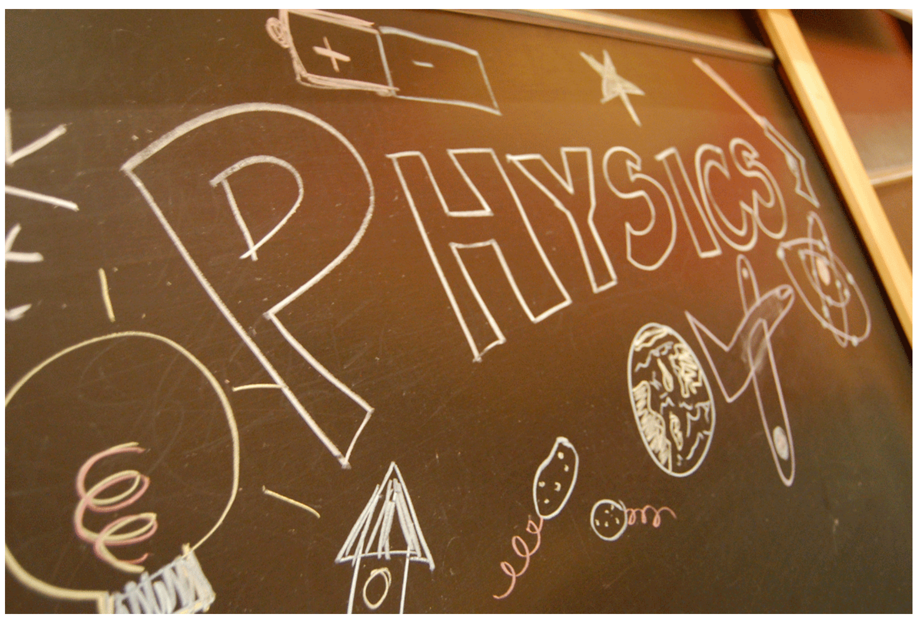
Abstract
It is an important part of learning that a person sees and engages a concept several times before mastery is attained (Arons, 1997). An example is the inverse square law. Many concepts in the NYS physics curricula can be explained by the inverse square law. Here I present a collection of activities that can be used throughout the school year to allow students to revisit the inverse square law using Newton's law of universal gravitation, Coulomb's law, and apparent brightness. The activities presented are connected to their associated New York State Regents Standards.

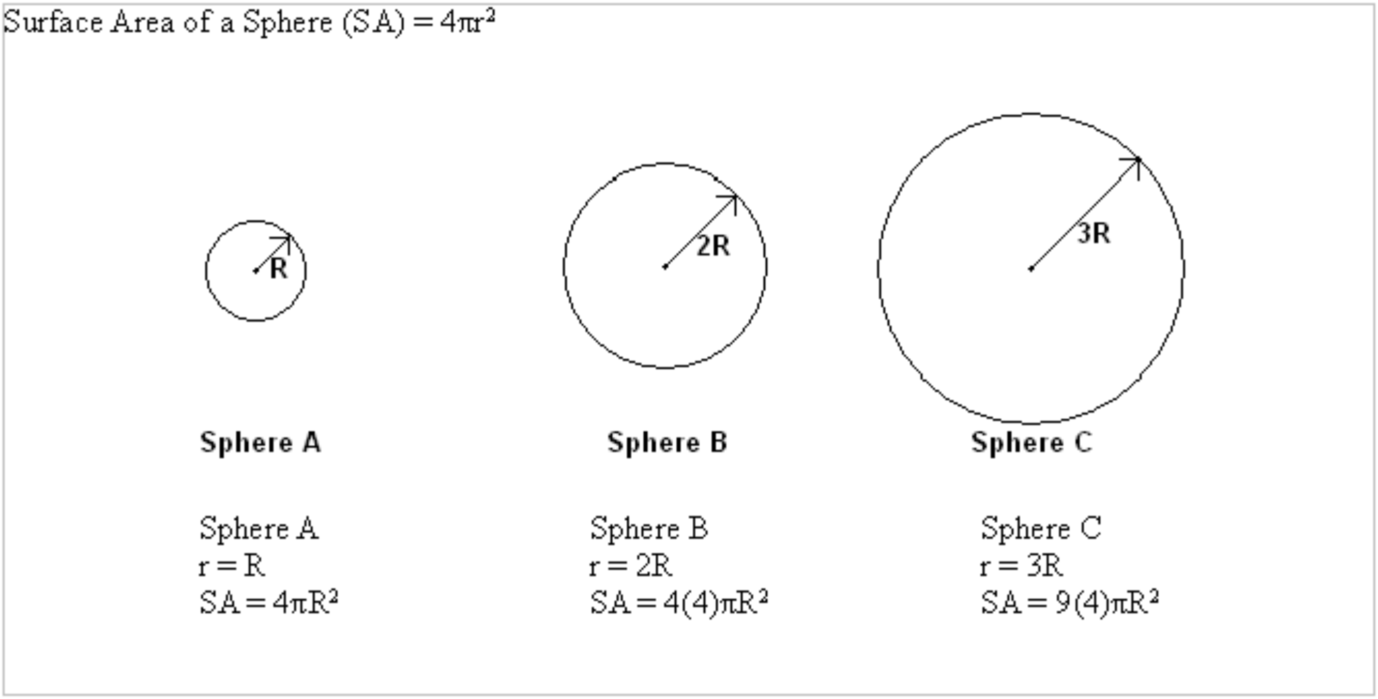
Introduction



Here I present a compilation of inverse square law activities that allow students to experimentally prove the relationship to themselves for various physical systems. These activities allow students to visualize the effects of the inverse square law in areas such as gravitational forces, electrostatic forces, and the apparent brightness of light sources. These activities have been chosen to provide "non-threatening" methods for teaching the inverse square law in a high school physics classroom, but they can easily be adapted for upper or lower level classes as needed.

Problem: The inverse square law explains many physical phenomena. The inverse square law's discovery is rooted in Isaac Newton's equation for universal gravitation, but it has been extended to other situations including Coulomb's Law, radiation, the description of apparent brightness of light waves, and Gauss' Law.

Figure 1: Spherical Scaling



The inverse square law is a common concept presented on final examinations in physics. For example, a question from a New York State Regents Physics examination (June 2009, p. 3, question 10) was (http://www.nysedregents.org/physics/20090624exam.pdf):

A distance of 1.0 meter separates the centers of two small charged spheres. The spheres exert gravitational force F_g and electrostatic force F_e on each other. If the distance between the spheres' centers is increased to 3.0 meters, the gravitational force and electrostatic force, respectively, may be represented as

Responses:

(1) $\frac{F_g}{9}$ *and* $\frac{F_e}{9}$ (3) $3F_g$ *and* $3F_e$

(2) $\frac{F_g}{3}$ *and* $\frac{F_e}{3}$ (4) $9F_g$ *and* $9\ F_e$

When students look at this problem there are many things they could misunderstand when reading it, but even if they do interpret the problem correctly, misconceptions about scaling cause many students problems. According to Arons (1997), "the great majority of students, including those in engineering-physics courses, have very serious difficulty with such questions…they have no idea what to do in the absence of formulas for the relevant areas and volumes." (p. 12)

The problem students have when studying equations related to Arons' statement is that students tend to memorize those equations without understanding what each means and how those equations affect their results. And since the students lack understanding, they have difficulty using this information for any problems other than the ones explicitly presented to them in their classrooms and books.

> It is illusory to expect to remediate these difficulties with a few quick exercises, in artificial context, at the start of a course. Most students can be helped to close the gaps, but this requires repeated exercises that are spread out over time and are integrated with the subject matter of the course itself. (Arons, 1997, p. 1)

As a result, it is important that the inverse square law be visited and revisited throughout a student's academic career and in various forms in order to expect them to understand it as well as is necessary for mastery.

Inverse Square Activities

Table 1: A Summary of Inverse Square Law Activities and Materials

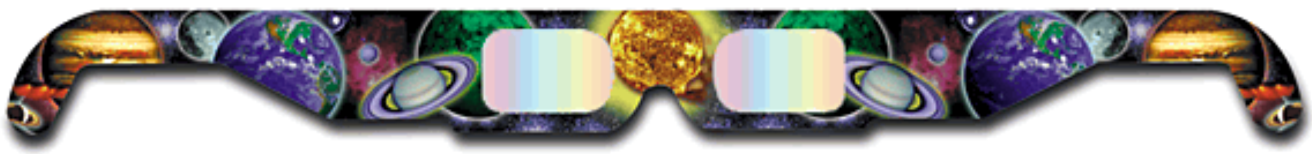
Subject Matter	Activity	Author	Materials	Estimated Time
Introductory Activities	A data analysis for the inverse square law	Downie	Projector Screen Graphing Materials	30 minutes
	Rainbow glasses and the inverse-square law	Doebler & George	Rainbow Glasses Monochromatic Laser Rulers Graphing Materials	40 minutes
	Unit 1 – worksheet 1:graphing practice	Hestenes & Wells	Graphing Materials	30 minutes
Gravitational Activities	Teaching universal gravitation with vector games	Lowry	Graph Paper Rulers	40 minutes
	Elliptical orbit: 1/r^2 force	Prentis, Fulton, Hesse, & Mazzino	Rulers Pins String	80 minutes
Coulomb's Law Activities	Demonstration of Coulomb's law with an electronic balance	Cortel	Electronic Balance Metalized Christmas Tree Balls Three Plexiglas Tubes Rulers Graphing Materials	30 minutes
	Coulomb's law on the overhead projector	Mahoney	Overhead Projector Two Conductive Balls Nylon String Wooden Support Block Wooden Dowel Graphing Materials	25 minutes
	E&M Unit 1 – mapping the electric field	Hestenes & Wells	Calculators Rulers	15 minutes
Wave-front Intensity Activities	Using a laptop screen to model point-source, line-source, and planar-source fields	Bohacek & Gobel	Laptop Black Construction Paper Rulers Photometer Graphing Materials	30 minutes
	Inverse-square law of light with Airy's disk	Narayanan & Narayanan	Monochromatic Laser Pinhole for Airy's Disk Rulers Photometer Graphing Materials	30 minutes

Introductory Activities

The simplest demonstrations of the inverse square law involve area measurements from a point-source of light. Two activities that use this method are *Data Analysis for the Inverse Square Law*, (Downie, 2007) and *Rainbow Glasses and the Inverse-square Law* (Doebler and George 1994). Additionally, the ASU Modeling Curriculum (Hestenes, 2006) also introduces the inverse square law as a part of its unit on Scientific Thinking, *Unit 1 - Worksheet 1: Graphing Practice* (modeling.asu.edu/Curriculum.html).

Downie's (2007) activity for the inverse square law involves the use of a slide projector or other light source with the ability to project an image. The image in this activity is a series of uniform dots aligned in a grid formation. The students measured how far the projector was from a screen with an area marked on it. While the projector was in this position, the students counted the number of dots that are in the marked area. Students repeated this process for various distances between the projector and the screen. Once the data was collected, the students plotted the data on a graph with axis of number of dots versus distance from the screen and came up with an inverse square relationship on their graphs. Students may not immediately know how to determine what the equation for this curve is, but that is dependent on the instructor's expectations, guidance, and the level of the students.

The activity presented by Doebler and George (1994) requires rainbow glasses, sometimes known as firework glasses. A company called Rainbow Symphony (http://www.rainbowsymphony.com/3dfrwrks.html) builds glasses made from a two-dimensional diffraction grating that causes light traveling through them to split into beams of light that form a grid. This grid forms a nearly perfect set of points to form a square on the screen in front of the glasses when a monochromatic laser is shone through them. The students assembled the laser and glasses setup and set it at various distances from a screen. The students measured the distance to the screen and determined the area of the square projected on the screen from each distance. The students then plotted the graph of area versus the square of the distance from the screen. Students can then draw a line of best-fit for and derive the equation of that line.



Rainbow Glasses:
From:http://www.rainbowsymphony.com/planet1.html

The ASU modeling activity, *Unit 1 – Worksheet 1: Graphing Practice*, is a graphical representation activity where students explore various data sets and use data analysis tools to determine the equations for the functions. After a brief introduction using notes and teacher guidance the students plotted graphs and visually analyzed the data to determine what action should be done to the graph axis to turn it into a straight-line graph. Some of the data sets in this activity allow students to see what a data set looks like for an inverse square relationship (Hestenes, 2006).

Each of these activities have strengths and weaknesses when being applied to the classroom. It is up to the instructor to determine which activity is best for their students' needs. The Rainbow Glasses activity makes mention of the cost effectiveness of the glasses and lasers, but needs to be explicitly mentioned as a useful tool for teaching the inverse square law. This link needs to be made because the data associated with the Rainbow Glasses activity implies that the distance from screen is proportional to the area of the square. This is not the inverse relationship the students are attempting to prove. Therefore the teacher must make the explicit connection that the area of the square is inversely proportional to the brightness of the screen at each distance. Downie's Dots and Projector activity provides a direct data set for the inverse square law but the activity requires the use of a projector for each group. The Graphing Practice activity is fast and efficient but provides no data collection activity which many students may need to effectively interpret the concept properly.

Gravitational Activities

Another place in the physics curriculum to reinforce the inverse square law is in the section for gravitational force and motion. This is a difficult topic to show students the connection because we cannot vary the gravitational force for easy experimentation in a classroom setting. However some teachers have made it possible for students to make measurements in their classrooms based on games and geometric activities.

One activity that creates a strong conceptual example of the gravitational relationship to the inverse square law is *Teaching Universal Gravitation with Vector Games* (Lowry, 2008). In this activity, Lowry takes the "Space Race" concept presented by Vinson (1998) and adds his own twist by creating a black hole on the game board. The black hole is incorporated by having students measure their "ship's" distance from the black hole. The students then squared that distance and used the information to apply the black hole effects to their ship. Lowry states, *that many of his students are surprised by the drastic differences in motion caused by the black hole and that they gain a strong conceptual understanding of the gravitational effects in relation to the inverse square law which can then be referred to during explicit instruction* (Lowry, 2008, p. 521).

A second activity that can be used to show the relation of gravitational effects was presented by Prentis, Fulton, Hesse, and Mazzino (2007). The goal of their Elliptical Orbit activity was to illustrate a geometric method to prove that the force of gravity on celestial objects is actually an inverse square relationship. Students laid out the typical orbit activity with two pins as the foci of the elliptical orbit and a string to mark the orbit of the "planet" around its star. The students then used Newton's assertion that for infinitesimal sections of an elliptical orbit, the path of the object is nearly parabolic. They then measured the deviation from the tangent of the object in its orbit. This deviation was compared to the time interval of that portion of the orbit. The time interval is based on the area of the orbit being swept out during that interval. After completing all of the measurements and calculations associated with the Elliptical Orbit activity, the students came to a result that showed that the gravitational force has an inverse square relationship.

Each of these activities has its mathematical and conceptual strengths. Both activities require students to use equations that involve the inverse square component to be successful and allow students to explore the deviation from normal everyday experiences. However, both of them do not illustrate realistic situations in order to accomplish their goals of teaching the inverse square law's effect on gravity. The Space Race activity accomplishes the task of effectively teaching the students by making it fun and enjoyable to learn. The Elliptical Orbit activity uses an extreme situation of a strongly elliptical orbit to generate data. But both seem to be useful in their arenas and are very effective for keeping the mathematical portion of the activity low while producing results the students can understand. Additionally, a website produced by the University of Colorado at Boulder (www.phet.colorado.edu) has many web applets that show promise in demonstrating gravitational motion and effects based on the inverse square law.

Coulomb's Law Activities

Two methods of introducing Coulomb's Law in the classroom were presented by Cortel (1999) and another by Mahoney (1971). Although both of these activities are a little older, they both present Coulomb's Law in a simple manner that also allows the teacher to introduce the inverse square law to their students. Another brief mathematical activity based on Coulomb's Law that also demonstrates the inverse square law can be found in the ASU Modeling Curriculum (Hestenes, 2006). The ASU activity is from the E&M semester materials, *UNIT 1 – Mapping the Electric Field* (modeling.asu.edu/Curriculum.html).

In Cortel's (1999) activity, the students used an electronic balance and two metallic spheres to show that the electrostatic force is inversely proportional to the square of the distance between their centers. It also allows those students to see the proportional relationship of force and charge. To focus on the inverse square nature of the activity, students set up a stationary platform with one of the metallic spheres attached to it on the balance. Students then set up the other sphere on a moveable ring stand hanging over the first sphere. The spheres were both charged with like charges and the students measured the force applied to the scale at various distances between the spheres. The students then plotted the data as force versus distance. Using data analysis tools, students can easily determine that the force is related to the inverse square of the distance between the objects.

Mahoney's (1971) activity is a class demonstration activity created for teaching students using an overhead projector. He set up a grid on an overhead projector with one conductive ball hanging from the top of the projector by some nylon strands and the other conductive ball was manipulated by being attached to a wooden block. Both spheres had an identical charge. This arrangement had the advantage that the entire class could see the spheres reactions and motions on the wall of the classroom, and everyone could take the measurements from the class example. The students measured two things: 1) the distance between the two balls on the screen, and 2) the displacement of the hanging sphere from its equilibrium position. The displacement of the hanging sphere was used as a substitute for the force between the spheres. Students in these classes graphed the relationship between the distance between the balls and the displacement "force" to discover that the relationship between them is the inverse square law.

Mapping the Electric Field is a brief activity that enables students to calculate the force on a test charge at various positions surrounding a central charged object using Coulomb's Law. After students calculate the forces, they then drew vectors to represent the forces at those locations. Students are often surprised to find out how much stronger the force is when they get close to the central charge. In this situation, the students are not deriving the inverse square law directly and are only discovering the effects of it. However this can still be a very useful activity for them. (Hestenes, 2006)

It is important to realize that with both the Electronic Balance activity and the Overhead demonstration that continuous charge leak will cause error in measurements if the equipment is not properly prepared. As described in the examples using area above, these Coulomb's Law verification activities provide teachers with both costly and inexpensive methods for teaching students the inverse square law at varying levels of accuracy. In these activities, it is interesting to note that the more expensive method requires fewer intellectual leaps to grasp the understanding as the measurements are already in the proper units. Though electronic balances are becoming less expensive, the degree of accuracy necessary may be a limiting factor in deciding which classroom balances you require to complete the activity. The overhead demonstration may take a little more time in class to explain that the distance moved by the dependent object is going to be proportional to the force between the objects. Depending on the level of the students, a proof of the relationship might also be necessary. The Mapping the Electric Field activity is not a hands-on activity, but does get the point across in a fast and efficient manner.

Wave-front Intensity

One last major area of introductory physics that the inverse square law can be emphasized is in the study of waves. Two activities presented here use light sources to demonstrate that the apparent intensity of the light from a point-source also follows the inverse square law. One of those activities was presented by Bohacek and Gobel (2011) and the other by Narayanan and Narayanan (1999).

In a Laptop Screen activity by Bohacek and Gobel (2011), students used a laptop screen to simulate a point-source of light. They measured the intensity of light hitting a light sensor at various distances from the computer screen. By plotting the data of intensity versus distance they determined that this is another inverse square relationship. The authors then took the activity one step further to show how a point-source situation is different from line-source or planar-source relationships. This may be a helpful connection to make to allow students to visualize examples of different fields that can be linked with the students' study of electric fields.

The Airy's Disk activity produced by Narayanan and Narayanan (1999) also makes use of a photometer to measure the strength of the light at various distances from its source. They accomplish this by shining laser light through a pinhole to produce Airy's disk. The intensity of illumination received by the photometer at various distances is then graphed in relation to the distance from the screen. An inverse square relationship is clearly presented. This activity calls for a photometer that can be set to detect a specific wavelength of light.

These wave-front intensity activities are easily connected to the area measurements examples given above. Both require more technology than some of the previous activities due to the nature of light and the technology necessary to make these measurements. But these activities provide data that is in the appropriate units and that fit nicely into the equation for the inverse square law and apparent brightness. There are no leaps of intuition or difficult calculations to make. The Airy's Disk activity might require more explanation and more expensive equipment in a classroom setting, due to the nature of Airy's disk and the need for a higher precision photometer.

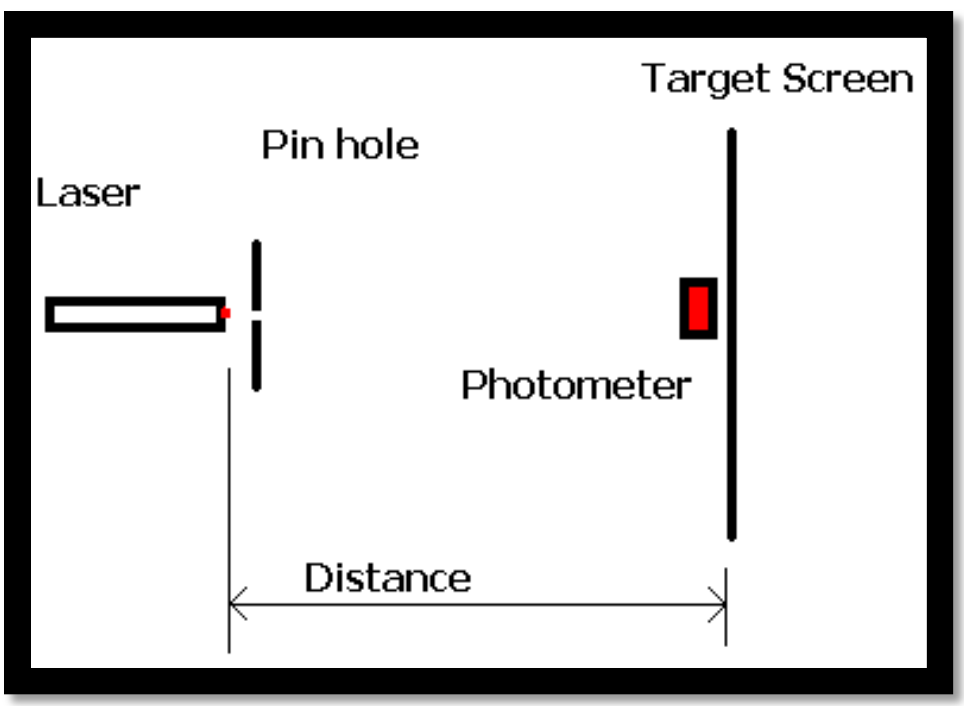


Figure 2: Graphic of Airy's Disk Experiment

New York State Curriculum

Due to the inquiry nature of these activities, they can easily be modified to a variety of grade levels. The major connections will be explicitly linked below, but for a full list of related NYS Regents Physics standards see Table 2.

Table 2: Correlations Between New York State Core Physics

Standards	Articles									
	A data analysis for the inverse square law	Rainbow Glasses and the Inverse Square Law	ASU – Modeling Activity: Unit 1 – Worksheet 1	Teaching Universal Gravitation with Vector Games	Elliptical Orbit → 1/r/r Force	Coulomb's Law on the Overheard Projector	Demonstrations of Coulomb's Law with an Electronic Balance	ASU - Modeling Activity: E1 – Mapping the Electric Field	Using a Laptop to model point-source, line-source, and planar-source fields	Inverse Square Law of Light with Airy's Disk
Standard 1 Key Idea M1.1	X	X	X	X	X	X	X	X	X	X
Standard 1 Key Idea M2.1	X	X	X			X	X		X	X
Standard 1 Key Idea M3.1		X	X			X			X	
Standard 1 Key Idea S1					X			X		
Standard 1 Key Idea S3.2			X		X		X		X	
Standard 6 Key Idea 3.2								X		
Standard 6 Key Idea 5.1	X	X	X		X	X	X		X	X
Standard 6 Key Idea 5.2	X	X	X		X	X	X		X	X
Standard 4 Key Idea 5.1v					X			X		
Standard 4 Key Idea 5.1viii				X	X					
Standard 4 Performance Indicator 5.1f				X	X					
Standard 4 Performance Indicator 5.1i				X	X					
Standard 4 Performance Indicator 5.1k				X	X					
Standard 4 Performance Indicator 5.1s				X	X			X		
Standard 4 Performance Indicator 5.1u	X	X	X	X	X	X	X	X	X	X

Since the format of almost all of these activities involve a laboratory exercise and graphing of some sort, these activities share many common NYS Physics Standards and NYS Mathematics Common Core Learning Standards. One of the most notable is Standard 1 – Mathematical Analysis Key Idea M1.1. This standard states that students should "use algebraic and geometric representations to describe and compare data." This standard is shared by every activity presented and provides a framework for allowing students to develop their own models of the observed phenomena. The other standard that quite obviously links all of these activities is Standard 4 – The Physical Setting Performance Indicator 5.1u which states that students should understand that "the inverse square law applies to electrical and gravitational fields produced by point sources." This is a clear indication of the importance of this concept in the NYS curriculum (University of the State of New York State Education Department, 2011).

Additionally, many other standards are shared between these activities but a few stand out as being more strongly shared. Two of them are further associations with Standard 1 – Mathematical Analysis, Key Ideas M2.1 and M3.1. A third strongly represented connection is Standard 6 – Patterns of Change Key Idea 5.2. These three standards are shared by more than 75% of the suggested activities and further represent the commonality of graphical and algebraic representations as well as the recognition of patterns that develop in the collection of data (University of the State of New York State Education Department, 2011).

Conclusion

These activities were chosen based on my experiences within the classroom and what I would like to try to use for my classes in the future. The strengths and weaknesses of the activities reviewed are compared in Table 3 (page 41). I have only previously tried the two activities associated with the ASU Modeling Curriculum and I feel that they have both strengthened my students' understanding of the inverse square law. I feel that investigations of the other activities presented here will greatly increase a student's understanding of the inverse square law.

Most students cannot understand a concept like the inverse square law without repeatedly visiting the content, addressing misconceptions and applying their understanding to new material. The activities presented here are designed to help students along the path to understanding and assist in retaining the concepts after instruction. They provide students with a variety of representations and various situations so they can make the connections and appreciate the commonalities between these phenomena. The use of these activities is of course at the teacher's discretion, and you may find that certain activities benefit your classroom better than others. But be sure to revisit the concept and make those connections for your students to be successful.

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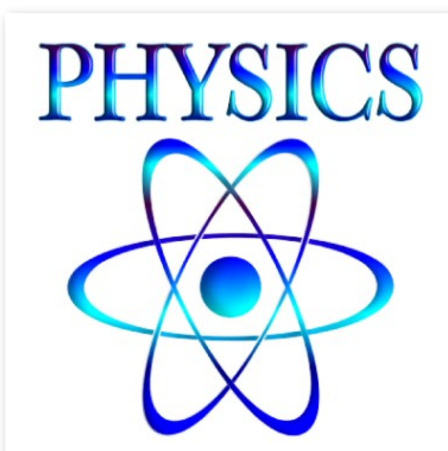

Table 3: Strengths and Weaknesses of the Reviewed Activities

Subject Matter	Activity	Strengths	Weaknesses
Introductory Activities	A data analysis for the inverse square law	Minor explanation required Data is easy to interpret	Several projectors are needed
	Rainbow glasses and the inverse-square law	Very inexpensive	Requires extra explanation from the teacher
	Unit 1 – worksheet 1:graphing practice	Very inexpensive Data is easy to interpret	No data collection activity
Gravitational Activities	Teaching universal gravitation with vector games	Very inexpensive Strong conceptual gains	No data collection activity
	Elliptical orbit: 1/r^2 force	Very inexpensive	Might require some geometry review of the subject matter
Coulomb's Law Activities	Demonstration of Coulomb's law with an electronic balance	Minor explanation required Data is easy to interpret	Balances may be costly Charge leakage is a concern
	Coulomb's law on the overhead projector	Very inexpensive	Not a hands-on activity Requires extra explanation from the teacher
	E&M Unit 1 – mapping the electric field	Very inexpensive Minor explanation required Strong conceptual gains	No data collection activity
Wave-front Intensity Activities	Using a laptop screen to model point-source, line-source, and planar-source fields	Data is easy to interpret Minor explanation required	Requires materials that can be difficult to get for a whole class
	Inverse-square law of light with Airy's disk	Data is easy to interpret	Requires materials that can be difficult to get for a whole class

References

Arons, A. B. (1997). *Teaching introductory physics*. NY: Wiley.

Bohacek, P. H., & Gobel, R. (2011). Using a laptop screen to model point-source, line-source, and planar-source fields. *The Physics Teacher*, 49, 124-126.

Cortel, A. (1999). Demonstration of Coulomb's law with an electronic balance. *The Physics Teacher*, 37, 447-448.

Doebler, R., & George, S. (1994). Rainbow glasses and the inverse-square law. *The Physics Teacher*, 32, 110-111.

Downie, R. (2007). A data analysis for the inverse square law. *The Physics Teacher*, 45, 206-207.

Hestenes, D., & Wells, M. (2006). *Modeling Instruction in High School Physics*. http://modeling.asu.edu/Curriculum.html

Lowry, M. (2008). Teaching universal gravitation with vector games. *The Physics Teacher*, 46, 519-521.

Mahoney, J. (1971). Coulomb's law on the overhead projector. *The Physics Teacher*, 9, 282.

Narayanan, V. A., & Narayanan, R. (1999). Inverse-square law of light with Airy's disk. *The Physics Teacher*, 37, 8-9.

Prentis, J., Fulton, B., Hesse, C., & Mazzino, L. (2007). Elliptical orbit: 1/r^2 force. *The Physics Teacher*, 45, 20-25.

University of the State of New York State Education Department. (2011). *Physical setting/physics core curriculum.* Retrieved May 18, 2011 from http://www.p12.nysed.gov/ciai/cores.html#MST .

University of the State of New York State Education Department. (2009). *Physical setting/physics regents examination.* June, 2009. Question 17. Retrieved from http://www.nysedregents.org/physics/20090624exam.pdf .

Vinson, M. (1998). Space race: A game of physics adventure. *The Physics Teacher*, 36, 20-21.

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