David Abbott's thoughts concerning David Rheam's PHY690 draft (5/27/08)

Overall Impression

This first draft is a great starting point (20 pages with lots of ideas!), but there's a lot of work to be done. The draft

1. Identifies a possible cause for student difficulties with exponential growth (overgeneralization of linear functions to systems with exponential growth)
2. Identifies examples of systems showing exponential growth and decay, including
   - Paper folding, rice on the checkerboard
   - Dice and Radioactive decay
   - Capacitor discharging, Container draining, Compound interest
3. Suggests activities for classroom use
4. Shows a concrete way to think about $e \stackrel{\text{def}}{=} \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$

Each one of the topic points listed is appropriate, but each of the topics needs to be explored in much greater depth. (I'll make some suggestions in the next section).

The draft does not yet feel like scholarship. It has more the feel of a term paper. The resources cited in the paper are inadequate: four TPT articles for a 20 page draft, supplemented by a handful of non-academic web pages which are quoted at length. The paper needs more original contributions/analysis from the author (rather than external sources).

There are a few minor mechanical things that need to be fixed when another draft is written. "Deep-seated" is misspelled throughout. Numbers in the text are not treated well. Numbers less than ten should be written as words; larger amounts, like 50, can be written as numbers. Web resources need to be properly cited (see specific comments in paper).

General Comments about Content Specific Matters

1) The draft suggests that students over-generalize linear models to describe systems which grow exponentially. No evidence for this assertion is given.
   - My experience teaching exponential growth does not support this contention. Students have lots of problems with the math, but, by and large, they do not seem to think that linear math will do the job (though sometimes they revert to linear math in calculations).
   - A survey of the literature on student difficulties should be done. What does the mathematics education literature have to say about student difficulties with exponents and exponential growth/decay? I know there's substantial stuff "out there" about the "familiar four" but I don't know what the literature has to say, for instance, about student difficulties with exponents (let alone about exponential functions).
2) The systems identified in the paper fall into distinct categories, each with slightly different behavior: those with a discrete "time" variable (e.g. rice, paper folding, rolling dice) and those with an essentially continuous "time" variable (e.g. radioactive decay, container draining,

capacitor discharge). Systems also have varying degrees of discreteness on the “population” variable, from essentially continuous for container draining and capacitor discharge to noticeably discrete for “radioactive decay” of a sample of dice. Some processes are stochastic (dice, radioactive decay, animal population models) and others are not. These system features lead to non-zero differences in behavior of the systems and may also pose different challenges for students. These differences should be acknowledged in the paper, if not explored.

3) The activities suggested in the paper are cool. I especially like the paper folding. However, what the students are supposed to learn from them is not articulated. What can the student expect to be able to do after the activity that the student was unable to do/recognize before? At a minimum, each lesson plan should have clear educational objectives. I think it would help to identify a hierarchy or progression of mathematical skills/tools/ideas that the student can be expected to develop.
4) The last section about e is problematic. It shows that e is not a simple thing. (Try coming up with a single sentence that summarizes the section!) The section shows a derivation but does not show how students might expect to be able to make use of the information. Why learn about e? The number e certainly is important in exponential growth/decay math- it is the essential link between discrete compounding and continuous compounding- but I’m not sure WHEN the appropriate time for students to learn about it is. I’m pretty sure that they do not need to learn about it until it is useful (that is, until it helps the students develop other understandings).