Using Gravitational Analogies to Introduce Elementary Electrical Field Theory Concepts

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Since electrical field concepts are usually unfamiliar, abstract, and difficult to visualize, conceptual analogies from familiar gravitational phenomena are valuable for teaching. Such analogies emphasize the underlying continuity of field concepts in physics and support the spiral development of student understanding. We find the following four tables to be helpful in reviewing gravitational and electrical comparisons after students have worked through hands-on activities analyzed via extended student discourse.[1]

Table I. Introductory analogies between gravitational and electrical forces.

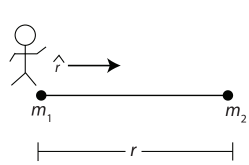
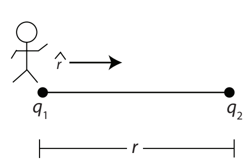
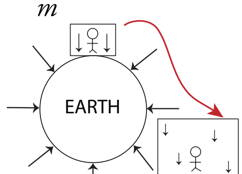
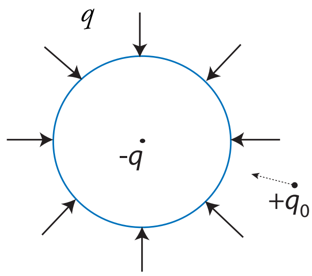
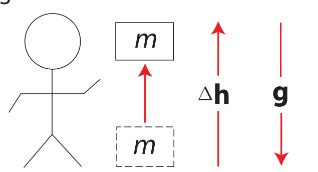
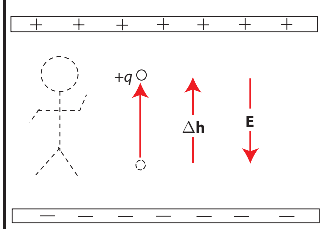
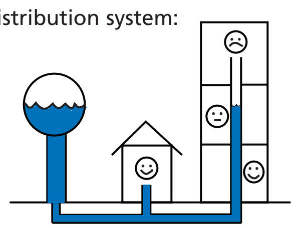
	Gravitational	Electrical	Comments
Forces: *Newton's Universal law of gravitation* and the *Coulomb law* for electric forces.	Matter has a fundamental property called *mass*, measured in kg, which has just one sign: positive. $\mathrm{F_g} = -G\dfrac{m_1 m_2}{r^2}\hat{r}$ describes the gravitational force and direction, where $\hat{r}$ is a unit vector describing the direction and negative means attractive. Gravitational force is therefore always attractive. The magnitude of this force is written: $\left\|\mathrm{F_g}\right\| = G\dfrac{m_1 m_2}{r^2}$ where in SI units: $G = 6.67 \times 10^{-11}$ N·m²/kg² 	Matter has another fundamental property called *charge*, measured in coulombs, which can have two signs: positive or negative. Hence electric forces can be repulsive or attractive. $\mathrm{F_e} = k\dfrac{q_1 q_2}{r^2}\hat{r}$ or in magnitude only: $\left\|\mathrm{F_e}\right\| = k\dfrac{q_1 q_2}{r^2}$, where in SI units: $k = 9 \times 10^9$ N·m²/C² 	Students may not know that so-called "antimatter" has positive mass (but reversed electric charges). These are point masses and charges or perfect spherical distributions of mass and charge. "Tinker toy" arrangements are later extended to real objects via calculus or symmetry. Some use the phrase "gravitational charge" for mass to exploit this analogy. Since G is much smaller than k, the gravitational force $\mathbf{F}_g$ is usually much weaker than the electrical force $\mathbf{F}_e$ (have students work both forces for 2 protons and 2 electrons and compare). Students may not yet be familiar with $\hat{r}$ (read aloud as r-hat) notation[2] but will need it in later physics. This notation is also used in discussing centripetal acceleration so review or introduce it. Note the tiny stick man in the figures defines $\hat{r}$ as a unit vector pointing to the other point mass or charge. $\hat{r}$ really contains direction information only. Notation requires lots of student practice and explicit explanation; use your state physics exam notation from the start of the course.

Table II. Introductory analogies between gravitational and electrical fields.

	Gravitational	Electrical	Comments
Vector Fields	For a small mass (compared to that of the Earth) on or very near the surface of the Earth, we can group together known terms and solve: $\lvert \mathbf{F}_g \rvert = G\frac{m_{earth} m_2}{r^2_{earth}}$ $= mG\frac{m_{earth}}{r^2_{earth}}$ then further group as $\lvert \mathbf{F}_g \rvert = m\lvert \mathbf{g} \rvert$ defining $\lvert \mathbf{g} \rvert \equiv \frac{Gm_{earth}}{r^2_{earth}}$, which is readily calculable, producing the famous |**g**| = 9.8 N/kg pointing down (toward the center of the Earth on the surface of the Earth). Now we can talk about the local field strength of the Earth's gravitation field at the Earth's surface, |**g**| being the ratio of the gravitational force on a "test mass" (a mass much smaller than that of the Earth very near the Earth's surface) to its mass. $\mathbf{g} = \frac{\mathbf{F}}{m}$  The gravitational field strength has units of force per unit mass or N/kg, which is the same as the more commonly used m/s^2. Field units are preferred, and we wind up explicitly re-stating: $\mathbf{F}_g = m\mathbf{g} =$ $m\,[-9.8 \text{ N/kg}]\,\hat{y}$, where $\hat{y}$ is a unit vector pointing upward. Now *g* should hold less conceptual mystery.	Similarly, with the electrical force there is a field around a given point charge *Q* (or spherically symmetrical distribution of charge *Q*), and it is useful to talk about the field strength around that charge. $\lvert \mathbf{F}_e \rvert = k\frac{q_1 q_2}{r^2} = q_0 k\frac{Q}{r^2}$ can be rewritten as $\lvert \mathbf{F}_e \rvert = q\lvert \mathbf{E} \rvert$ defining $\lvert \mathbf{E} \rvert \equiv k\frac{Q}{r^2}$. This is readily calculable for uniform electric fields—say, those very near a charged smooth spherical shell with charge *Q* or between two parallel plates with opposite charges as: $\mathbf{E} = \frac{\mathbf{F}}{q}$.  The corresponding units for the electric field strength are therefore force per unit charge or N/C, again with alternatively more common units of V/m (Table IV). An important value of |E| to know is |E| = 3 x 10^6 N/C or V/m — the dielectric breakdown strength of the Earth's atmosphere at STP. When this field strength is exceeded, air will be torn apart (ionized) and will conduct; we see sparks drawn through the air. Presence of electric sparks means we know an instant minimum value for |E|. [3]	We explicitly state the use of particular subscripts and capitalization for letters *m* and *q*, what is inferred in the use of each, and when and why we change subscripts. Although the universal law of gravitation formula will work with any two point or spherically symmetric masses, we most commonly experience the downward force of gravity at the Earth's surface. In that case one of the masses becomes the mass of the Earth and the distance is the radius of the Earth. Students perform this calculation of the gravitational field strength *g*. We walk around the class with a plumb bob—"a vanishingly small test mass m_0"—and compare the strength and direction of **g.** Note analogy to "a vanishingly small test charge q_0." First we stand on tables and then we hold the bob in different corners of the room, rudely determining by touch and vision that **g** doesn't measurably change in direction and size regardless of location. We start fields off with students sketching a figure (usually on a whiteboard) to explain the relationship between **g** in the classroom, **g** on the surface of the Earth at the equator and N and S poles, and **g** in space around the Earth. This develops a better understanding of **g** and makes explicit the **E** field analogy near both a point in space and near the surface of a charged shell like the dome of a Van de Graaff generator. We want to establish and reinforce the analogies between **E** and **g**. Stressing the units of **g** as N/kg helps to solidify the analogy when comparing to N/C for **E** (and can help clarify issues regarding gravitational fields). Students should show N/kg is equivalent to m/s^2, and later do the same for N/C and V/m. Also establish the similarity of **E** between two charged parallel plates[4] and **g** in a room on the Earth's surface. Parallel charged plates (e.g. aluminum pie pans) can be attached to a Van de Graaff generator to explore **E** with a packing peanut on a stick and thread or Christmas foil streamers. Also compare to the **E** near a Van de Graaff sphere. Students should memorize these particularly important numeric values of **g** and **E**, and be prepared to use them in discourse and on exams.

Tables III & IV. Introductory analogies between gravitational and electrical potential energy and potential.

	Gravitational	Electrical	Comments
Potential Energy	Gravitational potential energy is the stored energy associated with an object's mass attraction to other masses via a gravitational field. At the Earth's surface we find this by assuming a locally uniform field strength and direction: $\Delta PE_g \equiv -m\mathbf{g} \bullet \Delta\mathbf{r} = -m(-g)(+h)$, where **g** and Δ**r** are in opposite directions (lifting the object). Teachers should elicit via discourse how potential energy changes signs when displacement is in direction of, perpendicular to, or against the field.  	Electric potential energy can be found similarly, with more variations possible due to different possible signs of charge. $\Delta PE_e \equiv -q\mathbf{E} \bullet \Delta\mathbf{r}$ $\Delta PE_e = -q(-E)(+h) = qEh,$ when **E** and Δ**r** are in opposite directions (we also worry about the sign of the charge now).  	Electric potential energy is defined analogously to the gravitational potential energy discussed previously in the course. A topographic contour map should also be examined,[5] and the thought experiment of walking a wheelbarrow about contour lines or perpendicular to contour lines should be whiteboarded and discussed. Also the path taken by a loose ski or bowling ball free to fall down from a mountain peak on a topographical map. A useful potential energy analogy is stretching and releasing a rubber band to describe displacement with or against a field (and force). The role of path dependence should be explored in activities such as Arons' homework questions or the Modeling Physics worksheets.
Potential	Gravitational potential is the gravitational potential energy per unit mass, or for uniform gravitational fields: $\Delta V_g \equiv$ $\frac{\Delta PE_g}{m} = \frac{-m(-g)(+h)}{m} = gh,$ where the units are J/kg and object displacement opposite in direction to **g** (lifted) is assumed. The coined term we actually use for this is "liftage," somewhat analogous to "plumbing head"—where a scalar figure expresses where water can flow due to the use of a water tower in a water distribution system: 	Electric potential is defined as electric potential per unit charge or J/C, which turns out to be a very practical measure for electric phenomena: $\Delta V_e \equiv$ $\frac{\Delta PE_e}{q} = \frac{-q(-E)(+h)}{q} = Eh$ for displacement opposite in direction to the field. Electric potential is more commonly termed voltage (1 J/C = 1 V). A more common notation is $\Delta V_e = Ed$ for a positively charged object displaced antiparallel to the field.	When we talk about potential the analogy becomes less useful since in introductory physics we rarely discuss gravitational potential. The idea can be explained simply in terms of "liftage." Use the example of a water tower that holds the water for a municipality above the level of all the users' bathrooms. Therefore, the liftage is dependent on the height of the water, not the mass of the water. In other words, as long as there is water in the tower above the level of the bathroom, there can be water flowing in the bathroom. Potential can be a superior electrical descriptor compared to charge for conducting objects in contact. While identical conducting objects in contact share charge equally at equilibrium, conducting objects with different geometries do not share charge equally, though they do share electric potential at equilibrium.

Using These Tabulated Analogies

We have used this tabulated comparative approach in several courses, and have found the *least* successful way for students to learn these ideas is by presenting the complete tables in an early formal lecture, although students prefer such. Rather, we suggest that the ideas be formally presented in tabular form only *after* students have struggled with appropriate concrete hands-on activities, worksheets, and extended discourse both examining electrical phenomena and reviewing gravitational ideas.[6] The tabulated ideas could also be presented as final formal review notes, though in our experience students are unhappy with long delayed presentation of these formalisms. Our preferred balance is to reconstruct the information in each table in turn, "just in time" while moving through the subject as part of teacher-led "mini-lectures" or summaries encapsulating and formalizing student language and ideas. The idea is to reinforce student-negotiated language, meaning, and ideas developed via discourse[7] while moving toward standardized language and formalisms when students are ready for the formalism. Students frequently require reassurance that their ideas are correct, legitimate, and important (e.g. students might talk about electric contour lines later formalized as "equipotentials"). Hence, each of these four tables usually appears in our student notes as summary interludes among other activities. Formalizing pieces only at intervals when students are ready for such (and are requesting such) works well for us. One sequence we have followed (with many turnings and reorderings to suit student directions) has looked like this:

- Qualitative hands-on exploration of electrostatics (sticky tape, balloons, and foil). Chabay and Sherwood[7] have the most highly developed discussion of this area. Modeling physics has a nice electrophorus/Ne bulb activity we usually include here (or later; see below).
- Coulomb's law quantitative experiment (digitized modeling videos if necessary). Review with Table I.
- Invoke similarities to action-at-a-distance gravitation and re-examine gravitation both near and far from Earth's surface. Explore *g* and *E* fields with plumb bobs and packing peanuts on thread or Christmas foil streamers. Calculate and describe *g* and *E*. Review with Table II.
- Plot *E*-field-like phenomena using conductive paper or water in glass cake pans. Compare with topographical maps. Complete Modeling Physics worksheets and activities or someting similar. Table II.
- Use student discourse to develop need for terms such as *voltage* and *liftage*. Introduce Drude atomic-level model[7] for later use in dc circuits and illumination of electricity as water analogy shortcomings by refocusing on role of *E* field.[2,3] Review with Table IV.

Our approach works not only to help students develop meaningful and more concrete insights into these abstract electrical concepts, but also to reinforce and enrich their gravitational models. Instructional time is so limited with each individual topic there is very little opportunity for repetition, so this is an opportunity to spiral back on gravitation while leading into the use of atomic-level models describing current flow in circuits, the next portion of the curriculum.

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References

1. For a compelling vignette demonstrating advanced student discourse in the introductory physics class, see D. Desbien, M. Joshua & K. Falconer, RTOP Video 4 (SUNY-BSC, Buffalo, NY, 2003). Streamed QuickTime video available from http://physicsed.buffalostate.edu/rtop/videos/RTOP4/RTOP4play.html.
2. R. Knight, *Physics for Scientists and Engineers with Modern Physics: A Strategic Approach* (Pearson Addison Wesley, San Francisco, 2004), p. 808 develops the r-hat notation. The workbook is an excellent source of PER-inspired activities.
3. R. Chabay and B Sherwood, *Matter & Interactions Vol. II: Electric and Magnetic Interactions* (Wiley, Hoboken, NJ, 2002). This is an outstanding PER-influenced curriculum for studying electricity and magnetism. Section 14.8 is a case study of sparks in air. See the text website, http://www4.ncsu.edu/~rwchabay/mi, and their paper

R. Chabay and B. Sherwood, "Restructuring the introductory electricity and magnetism course," *Am.J. Phys.* 74 (4), 329–336, (April 2006)

4. A. Arons, *Teaching Introductory Physics* (Wiley, Hoboken, NJ, 1996). See problems 5.11, p. 73 (the charge ferry) and 5.17, p. 76 (parallel plate E field) of Vol. II in this three-section book.
5. B.S. Andreck, "Using contour maps to teach electric field and potential," *Phys. Teach.* **28**, 499 (Oct. 1989).
6. D. MacIsaac and K.A. Falconer, "Reform your teaching via the Reformed Teacher Observation Protocol (RTOP)," *Phys. Teach*. **40**, 479 (Oct. 2002); http://PhysicsEd.BuffaloState.Edu/RTOP.
7. See Ref. 3.
8. D. Hestenes, G. Swackhamer, and L. Dukerich, *Modeling Physics Curriculum* (ASU, Tempe, AZ, 2004); http://modeling.asu.edu/Curriculum.html. See second semester (electricity & magnetism) curricular materials (worksheets, digitized videos and activities), particularly Units E1: Charge and Field and E2: Potential. A freely reproducible HS curricular resource based on physics education research (PER) touchstone curricular activities.
9. P. Laws, *The Workshop Physics Activity Guide Module 4: Electricity & Magnetism* (Wiley, Hoboken, NJ, 2004). For a comparative development of *E* and *g* fields, see especially Unit 21: Electrical and Gravitational Energy, p. 573–593.

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The "Not So Inert" Noble Gases

"In 1894 Lord Rayleigh and Sir William Ramsay announced to the British Association the discovery of a new chemically unreactive element in the atmosphere that they named Argon, 'the lazy one.' Ramsay went on to identify the other gaseous elements Neon ("new"), Krypton ("hidden"), and Xenon ("alien" or "stranger") so that a new column of elements must be added to Mendeleev's Periodic Table. He and Rayleigh were awarded the Chemistry and Physics Nobel prizes, respectively in 1904 for their discoveries."

These gases were long thought to be chemically inert.

"That all changed in 1961, when Bartlett (no relation) first noticed that the ionization energies of O_2 and xenon were nearly identical, and he then prepared the first xenon salt (originally formulated as XePtF6). Shortly afterward XeF_4, XeF_2, $XeOF_4$ and XeO_3 were synthesized;... The beginnings of analogous krypton chemistry have also been described, although no hints of true argon or neon chemistry have been detected to date. The noble gases are definitely not chemically inert species."[1]

1. C.E. Kohlhase, "To Neptune and Beyond," *Science Digest* (Jan. 1982), p. 46.