A Hands-on Introduction to Displacement / Velocity Vectors and Frame of Reference through the Use of an Inexpensive Toy

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Abstract: This paper presents a set of hands-on activities used by the author with 93 students as an introduction to vector terminology and those vector operations common in the New York State Regents Physics curriculum (NYSED, 2008) with a focus on displacement and velocity vectors. Through guided activity worksheets (Appendices A and B) and the use of inexpensive equipment, students were able to visualize the tip-to-tail method of vector addition, determine the horizontal and vertical components of vectors and observe the combination of two concurrent parallel or perpendicular vectors. Students observed the motion of a wind-up toy on a moving Cartesian grid within a static frame to establish the concept of frame of reference for relative motion. The terminology and level of difficulty were focused toward a high school Regents class.

Biography: Gwen Saylor lives in the Hudson Valley area of New York. She received her B.A. in Biology from University at Albany in 1995. She worked as an educator in settings which range from outdoor education centers, lecture halls and private boarding schools before becoming a certified biology teacher in 2003. In 2006, she transitioned to teaching physics and began her work on a Masters in Physics Education which culminated in this project. She is currently a full time teacher at Arlington High School, from which she graduated in 1991.

Introduction:

Vectors are the natural language of mechanics. The activities presented in this document use a *Never Fall™* wind-up toy to create a hands-on activity for introducing vectors to Regents Physics students with little to no prior exposure to vector quantities. The introduction of vector quantities and vector operations were limited to displacement and velocity scenarios. The skills introduced through these activities will subsequently apply to the topics of projectile motion, superposition of forces, momentum and force fields.

The two activities presented in this document, *Activity One: Ladybug Transit* (Appendix A) and *Activity Two: Ladybug on a Conveyor Belt* (Appendix B) were created by the author to serve as instructional tools that make vector characteristics both explicit and highly visual for learners. The activity expands a teacher directed demonstration by Mader and Winn (2008) into a student centered activity. Each activity was designed to be conducted in the space of a student desktop.

Vectors in the New York State Regents Physics Curriculum

The following chart (see Table 1) summarizes the portions of the *Standards of Mathematical Analysis and Scientific Inquiry* that relate to vectors in the *New York State Physics Core Curriculum.*

Table 1: Vector Skills From the NYS Physics Core Curriculum	
Standard 1: Mathematical analysis	
Key Idea 1: Abstraction and symbolic representation are used to communicate mathematically - use scaled diagrams to represent and manipulate vector quantities	
Standard 4: Scientific Inquiry	
Key Idea 5: Energy and matter interact through forces that result in changes in motion.	
5.1a	Measured quantities can be classified as either vector or scalar.
5.1b	A vector may be resolved into perpendicular components.
5.1c	The resultant of two or more vectors, acting at any angle, is determined by vector addition.

(NYSED, 2008) Full text available at http://www.p12.nysed.gov/ciai/mst/pub/phycoresci.pdf

In order for a student to transition from the basic level of functionality listed in Key Ideas 5.1a-c in Table 1, toward mastery of skills and concepts in the remainder of the curriculum, learners must be able to demonstrate the following skills and understandings:

- Define terms such as displacement, velocity, resultant, equilibrant and component.
- Establish the relationship between component vectors and the resultant vectors including the concept of additive inverse (Arons, 1997, p. 107).
- Define the meaning of a negative vectors in relation to the horizontal and vertical axes
- Understand that vector quantities are not fixed to a location (Brown, 1993).

Background:

The *Vector Knowledge Test* (Knight, 1995), administered to introductory college level physics courses comprised of primarily science majors, revealed that nearly half of the students who self-reported prior exposure to vectors from high school physics or math entered the class with no useful knowledge of basic vector skills. Based on interviews and activities, Aguirre (1998) concluded that students commonly held misconceptions regarding vectors include the following:

- Speed and displacement are independent of frame of reference.
- Vector components act sequentially rather than simultaneously.
- Time is different for the resultant path than for the components.
- Magnitude of component vectors change when two vectors interact.

Knight's (1995) recommendations from his analysis of the *Vector Knowledge Test* (Knight, 1995) suggested that vectors should be introduced over a course of several weeks, prior to introduction of projectiles or Newtonian mechanics. Subsequent investigations using diagnostic testing of introductory college students noted that students demonstrated some intuitive knowledge of vectors but lacked the ability to apply skills such as tip-to-tail and parallelogram methods of vector addition (Nguyen and Meltzer, 2003).

From a student's viewpoint, "adding velocity arrows appears very different from adding displacement arrows, and acceleration arrows are totally incomprehensible" (Arons, 1997, p. 107). In survey of introductory physics students, graduate students and physics TA's, Shaffer and McDermott (2005) found that the ability to correctly draw and label a vector was markedly greater for velocity related concepts than for acceleration concepts. As instructors transition from displacement vectors to force vectors, students are likely to become confused unless the nature of each of these quantities is discussed (Roche, 1997).

A number of activities are widely used to introduce vectors to students. Vector treasure hunts are a popular method. In this type of investigation students use a compass to create a treasure map using vectors (Windmark, 1998). The map is then passed to another group for them to follow. This method requires prior knowledge of tip-to-tail addition. A force table is a common introductory experience used to teach vectors, mechanical equilibrium and the vector triangle (Greensdale, 2002).

Required Student Prior Knowledge

These activities are intended to be sequenced within the curriculum just after the introduction of the terms: displacement, velocity, vector and scalar. Basic vector terminology, such as resultant, equilibrant, horizontal and vertical components should be introduced at the outset of the activity.

Within the physics curriculum, vector operations are taught using both the Cartesian coordinate system (x,y) for the horizontal and vertical components and the polar coordinate system (R, θ) for the resultant vectors (Hoffmann, 1975). The origin of the polar coordinate axis is aligned with the positive "x" plane of the Cartesian coordinate system. The "R" serves as a symbol for any vector quantity, but displacement and velocity are substituted by students as appropriate. Cartesian and polar coordinate systems are not terms familiar to students, nor are they used in the Regents Physics curriculum. Therefore, the terms used here for Cartesian coordinate system values will be "horizontal and vertical components" which refer in equations to R_x and R_y respectively. Values reported in the polar coordinate system will be referred to by magnitude (R) and direction (θ) given in standard position or reference angle form as appropriate. To successfully complete these activities, students must be able to translate between these coordinates systems by applying the following transformation equations:

$R_x = R \cos \theta$ $R_y = R \sin \theta$ $\theta = \tan^{-1} (R_y/R_x)$ $R^2 = R_x^2 + R_y^2$

An understanding of methods used to express angles is required for reporting the direction of the resultants. Standard position refers to angles measured from the positive x-axis to the terminal side with respect to a 360 degree counterclockwise rotation (Ryan, 1993). For each angle of standard position students must be able to identify the reference angle and assign the appropriate quadrant. The reference angle is the acute angle formed by the terminal side of the given angle and the x-axis. For reference angles that do not fall in the first quadrant, students must be able to convert to standard position. Students must understand that the axes of the Cartesian coordinate system align with the quadrantal angles of 0^0, 90^0, 180^0, 270^0 and 360^0. For example, a polar coordinate vector of magnitude R at an angle of 180^0, would be written as $R_x = -R$, $R_y = 0$.

The table below lists the performance indicators for the NYS Regents math courses that cover content related to the required prior knowledge discussed here.

Table 2: *Integrated Algebra* (A.A.) and *Algebra 2 and Trigonometry* (A2.A) Performance Indicators	
A.A.42	Find the sine, cosine, and tangent ratios of an angle of a right triangle, given the lengths of the sides.
A.A.43	Determine the measure of an angle of a right triangle, given the length of any two sides of the triangle.
A.A.44	Find the measure of a side of a right triangle, given an acute angle and the length of another side.
A.A.45	Determine the measure of a third side of a right triangle using the Pythagorean theorem, given the lengths of any two sides.
A2.A.57	Sketch and use the reference angle for angles in standard position.

(NYSED, 2005) Full text available at http://www.p12.nysed.gov/ciai/mst/math/standards/core.html

Nearly all students enrolled in Regents Physics within our school have completed *Integrated Algebra*. Therefore, a basic understanding of trigonometric functions was assumed. Approximately eighty percent of my students are concurrently enrolled in *Algebra 2 and Trigonometry*. The remaining twenty percent of students are evenly split between *Geometry* and a higher level course. Given the composition of my classes it was necessary to provide students with some formal instruction on the transformation equations and standard position angles prior to these activities. The simulation, *Vector Addition*, available through PhET, was used to reinforce the connection between the two coordinate systems and allowed students to practice with the transformation equations (*http://phet.colorado.edu/en/simulation/vector-addition*).

Activities

Ninety-three students, enrolled in my three sections of 31 students each, completed both activities including worksheets. *Activity One* required roughly 80 minutes for introduction, student work and discussion. *Activity Two* required roughly 90 minutes for introduction, student activities and discussion. The completion of the follow-up questions required additional class time or were assigned as homework. Both of these activities took place within a science classroom setting as they require space and a flat desk or table.

Each activity utilized a guided worksheet but required active manipulation of materials. Students worked in small groups to problem solve throughout the guided activities. The guided format was necessary due to the fact that this was an initial introduction to vectors. Students would have experienced great difficulty creating the scenarios for themselves.

Equipment:

Never Fall™ wind-up toys were used because they met the requirements of both activities. In *Activity One* it was necessary for the toy to pivot when it reached the edge of a surface. In *Activity Two,* the toy required a low center of gravity to prevent tipping when the surface Cartesian grid was moved and needed to maintain a constant velocity for approximately six seconds. These toys come in several varieties and ladybug themes were selected for this class to add some levity for the adolescent audience. I also purchased and experimented with the bulldozer variety but rejected these due to a lower speed. *Never Fall*™ toys are widely available online and in toy stores for a cost of approximately $3-4 each. It is advisable to purchase extra toys to allow for malfunctions or breakage. The *Never Fall*™ wind-up toy will hereafter be referred to as the "ladybug toy."

The surface for both *Activity One* and *Activity Two* was a dry erase poster board with Cartesian grid lines. These are available at teacher supply stores. For *Activity One* the dry erase surface was attached to a piece of foam board with double sided tape to provide an edge (see Figure 1a). For *Activity Two*, the movable surface should be 50-60 cm in length and a minimum of 25-35 cm in width and have a dry erase finish. The lines are helpful to verify straight motion of the ladybug toy. An additional large dry erase board, three meter sticks and a stop watch are also required (see Figure 1b). The cost of each lab set-up, consisting of dry erase poster board and toys came to approximately $10 (not including stop watches and meter sticks).

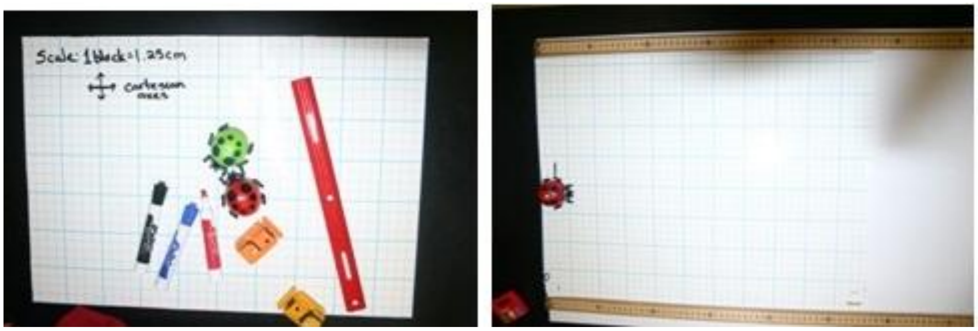


Figure 1a and 1b: Required equipment for activities

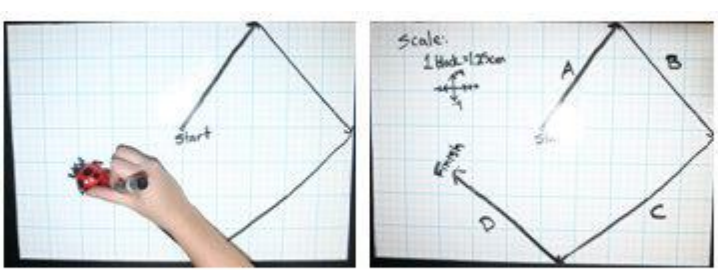
<u>Activity One: Ladybug Transit</u>

Overview:

Students traced the motion of the ladybug toy on a Cartesian grid and compared the path travelled with the horizontal and vertical components of the ladybug toy's motion. For each displacement vector, students applied the transformation equations to determine the relationship between the components (R_x, R_y) and the resultant (R, θ). *Activity One: Student Worksheet* (Appendix A) provided questions and data tables to guide and organize student work.

Procedure and Instructional notes:

A fully wound ladybug toy was released from a point close to the center of the board and the motion was traced with a dry erase marker (see Figure 2a). Trials in which the ladybug toy traveled a path made up of at least three distinct vectors were considered successful. Each vector was labeled with an identifying letter. Students determined a measurement scale in centimeters for each block of the Cartesian grid and labeled the direction for the horizontal and vertical components accordingly (see Figure 2b).



Figures 2a and 2b: Sample student work, steps A-D

The horizontal and vertical components of each vector were determined in block units (see Figure 3) and recorded in the data table (see Figure 4) using positive or negative signs to note the direction correlated with each axis. All displacements measurements were originally stated in block units since not all groups were utilizing surfaces with an identical scale. The scale was determined in centimeters and recorded by students.

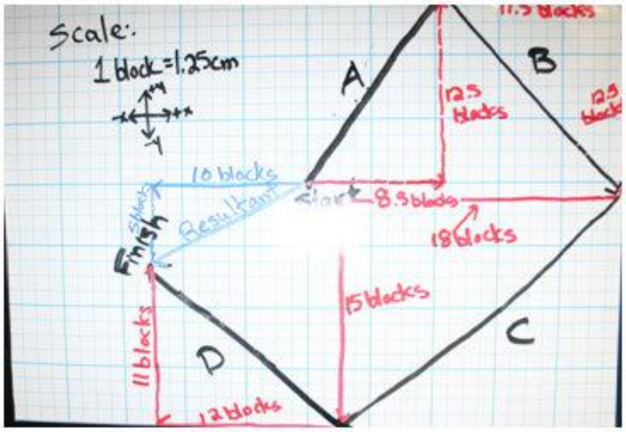


Figure 3: Sample of student work step E**.**

The total horizontal and total vertical displacements were found by adding each column. The resultant displacement was drawn as an arrow from the starting point to the end point. The horizontal and vertical components of the resultant vector were also found and compared with the results found by adding each column in the data table (see Figure 4).

Using this information, students determined the magnitude (R) and direction (θ) of each displacement vector. To determine the magnitude of each vector in block units the Pythagorean Theorem was used. The magnitude was then converted to centimeters. Reference angles were determined using the equation '$\theta = \tan^{-1}(R_y/R_x)$'. Students then found the angle in standard position based on the quadrant for the vector as determined by the horizontal and vertical components. The calculated magnitudes and angles were recorded in the data table to correspond with the displacement vectors traveled by the ladybug toy and the vector for the resultant displacement (see figure 4).

	Components		**Resultant**			
	Horizontal X (Block Units)	**Vertical Y** (Block units)	**Magnitude (block units)**	**Magnitude (centimeters)**	**Reference Angle**	**Angle (given Standard Position)**
A	8.5	12.5	15.1	18.9	55.8	55.8
B	11.5	-13.5	17.7	22.2	49.6	310.4
C	-18	-15	23.4	29.3	39.8	219.8
D	-12	11	16.3	20.3	42.5	137.5
∑	**-10**	**-5**	**11.2**	**14**	**26.5**	**206.5**

Figure 4: Sample data table

Students were then asked to compare magnitude and direction values found in the data table with the original displacement vectors traced on the board (see Figure 3). Students were asked to look at each vector independently, compare with the data table and determine results were reasonable and realistic. Discrepancies were discussed amongst the group.

As a final phase of the activity, students investigated how changing the order of vectors would influence the magnitude and direction of the resultant. In order to achieve this, students wrote resultant (R, θ) for each vector travelled by the ladybug toy onto a separate index card. Each group shuffled the cards to produce a random order of vectors and then exchanged cards with another group. Students were then tasked to find the resultant displacement of the other group's ladybug toy using steps learned during the activity. Results were then compared between groups and discussed. Students concluded from this phase of the activity that the order in which you add vectors does not influence the magnitude or direction of the resultant.

Activity Two: Ladybug on a conveyor belt

Overview:

Students explored the simultaneous motion of a wind-up toy and the surface on which it moved. The movable surface served as the "conveyor belt" upon which the ladybug travelled. On a stationary surface, the velocity of the toy is found by dividing the displacement by the time interval. When the surface upon which the ladybug toy is in motion the, the resultant displacement and velocity of each component must be determined relative the static frame. The displacement of the ladybug toy was measured in relation to the moving surface and against a set of fixed meter sticks. The motion of the Cartesian grid was always measured against a fixed frame of reference consisting of a dry erase board with meter sticks attached to a large dry erase surface and below the movable surface.

Students were able to use both the moving surface and the fixed dry erase board to record starting and end points, sketch displacement vectors and to solve for values requested in the student worksheet. Students recorded data and findings in the appropriate data table on the student worksheet.

A third meter stick and protractor were used when necessary to measure the vertical axis or measure resultant displacements at an angle. The time frame for the displacement was measured using a hand held stop watch.

Procedure and Instructional Notes:

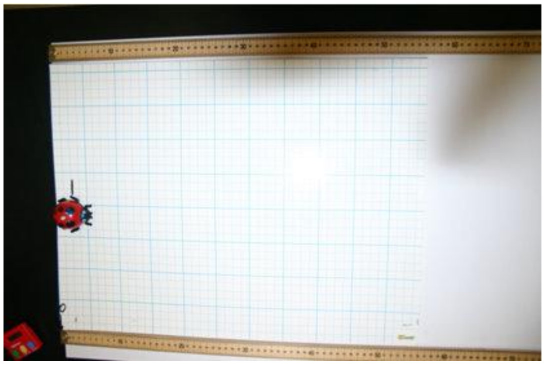
After a brief introduction to the equipment, students set up materials and divided tasks as follows.

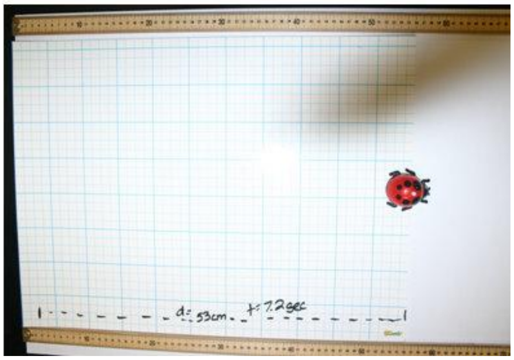
- One student was responsible for pulling the "conveyor belt" at constant speed.
- One student was responsible for timing the motion with a stop watch.

- One student was responsible for winding the ladybug toy before each release, and stopping the ladybug toy at the conclusion of the scenario. This individual was responsible for clearly communicating start and stop to the other members of the group.
- One student used a dry erase marker to indicate changes in displacement. The start and end points for the following values needed to be determined: 1) the change in position of the Cartesian grid relative to the fixed frame 2) the change in position of the ladybug toy relative to the Cartesian grid and 3) the change in position of the ladybug toy relative to the fixed frame. This task was assigned to the student responsible for timing the motion when groups consisted of only 3 students.

Students practiced manipulating the equipment and coordinating roles before completing scenarios and recording data on the *Activity Two: Student Worksheet*. Groups also discussed and agreed upon a common set of procedures for releasing the ladybug toy, stopping the ladybug toy and measuring results. For example, a common point on the ladybug toy was used for measuring all displacement values. Students also agreed the ladybug toy would be fully wound prior to each scenario. Since students standing around the table had different view points for observing motion, the groups each agreed to an orientation for horizontal and vertical axes. The Cartesian grid surface was able to move between the meter sticks on the horizontal to the right (+x) or to the left (-x). The ladybug toy was able toward the right (+x), left (-x), up (+y) and down (-y).

In the first stage of the activity students determined the average speed (or magnitude of the velocity) of the ladybug toy by averaging three trials. This was accomplished by allowing the toy to traverse the width of the stationary Cartesian grid and recording both displacement and time of motion (see Figures 5a and 5b).





Figures 5a and 5b: Determining speed of toy (before and after)

Once an average speed was established, students proceeded through a range of scenarios involving concurrent motions of the toy and the Cartesian grid. Each scenario demonstrated how the resultant motion of the ladybug toy relative to the fixed frame reference is dependent on the direction and magnitude of the component velocities.

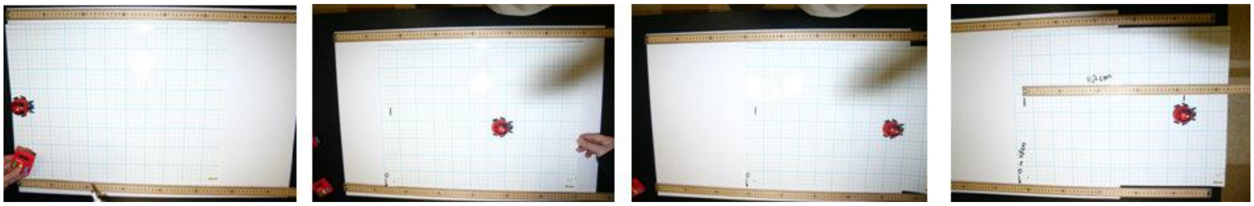


Figure 6a-d: Parallel velocities in the same direction (before, during, after).

Students compared the resultant displacements and velocities for parallel components moving in the same direction (see Figures 6a-d) with parallel components moving in opposite directions (see Figure 7a-b). When the velocities were oriented in the same direction the resulting displacement was large. When the velocities were oriented in opposite directions the resultant was small or zero.

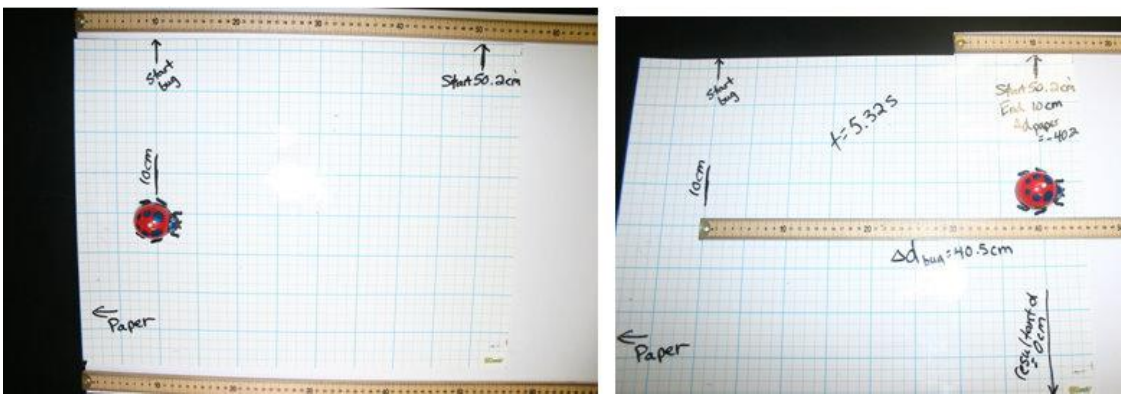


Figure 7a-b: Concurrent velocity in opposite directions, before and after.

For scenarios involving perpendicular components, students created a sketch of the path travelled as seen from a fixed frame of reference. The magnitude and direction for each combination of horizontal and vertical components were compared graphically (see Figure 8a-b).

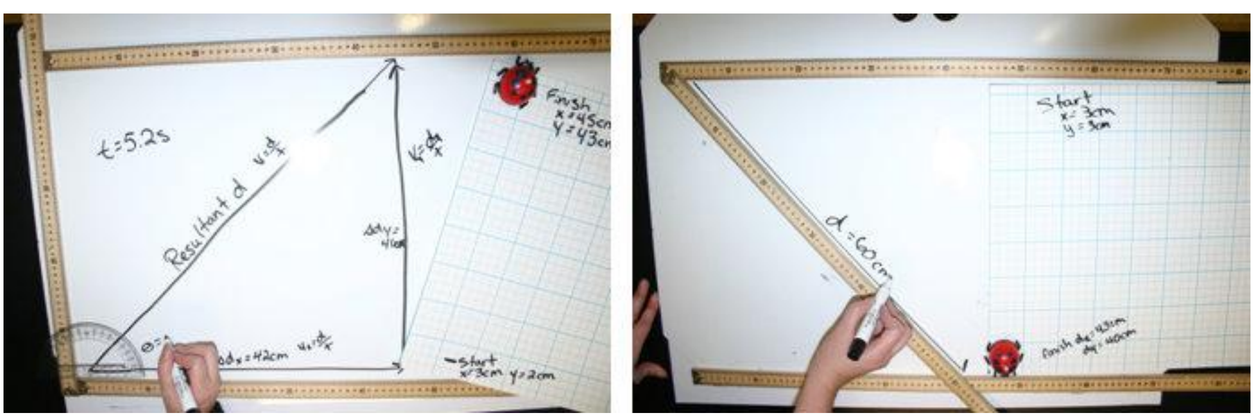


Figure 8a and 8b: Perpendicular motions of ladybug toy and Cartesian grid.

In each scenario, students were able to observe that a resultant displacement calculated by Pythagorean Theorem and a transformation equation were the same as the resultant as measured directly with a meter stick and protractor. By varying the speed at which the surface was pulled, students discovered how a change in the magnitude of the horizontal component influenced the magnitude and angle of the resultant displacement and velocity.

Reflections for *Activity One: Ladybug transit*

Students were able to easily break down the displacement vectors that served as the ladybug toy's path into horizontal and vertical components using the Cartesian grid system. Most students applied the positive and negative signs to the horizontal and vertical components on the appropriate axes with minimal difficulty with the presence of the Cartesian grid. The Cartesian grids helped address student confusion about the meaning of a negative R_x or R_y resulting from a transformation equation. Students were able to visualize tip-to-tail addition of vectors repeatedly throughout the activity. In subsequent class work, the majority of students were able to correctly draw a vector diagram showing perpendicular components and a resultant. Students were also able to use the transformation equations to find the magnitude and direction of the resultant given values for the components. The second phased of the lesson was successful in demonstrating to students that the order in which vectors are added does not change the magnitude or direction of the resultant.

Reflections for *Activity Two: Ladybug on a Conveyor Belt*

This activity introduced the idea of concurrent vectors but also further developed the concepts of displacement and velocity. As I walked around the room I heard many student conversations in which groups were actively working through the difference between the concepts of displacement and velocity. In several cases I needed to work with students on the difference between the two values. This was very insightful. The difference between how far an object travels and how fast an object travels seems implicit.

The activity provided a useful visual for students to establish a model for future discussions. In particular this activity emphasized the importance of the concept of frame of reference. The method I used to help students understand the idea of frame of reference was to query students as to what would be observed from various view points. Examples of queries included: What would a ladybug toy moving parallel on the vertical path to the first ladybug observe? What would a ladybug moving only due to the horizontal motion of the paper observe? How are these motions different from a ladybug watching from a fixed position near the starting point? After adequate discussion students were able to explain the role of frame of reference in determining the magnitude and direction of the resultant.

The activity achieved the stated objectives. Students confronted each of the common misconceptions regarding vector quantities outlined by Aguirre (1998). The following table outlines the misconceptions and the strategy utilized within the activity to address the misconception.

Table 3: Vector Misconceptions (Aguirre, 1988) addressed in *Activity Two*: *Ladybug on a Conveyor Belt*

MISCONCEPTION	ACTIVITY CONNECTION
Path is an intrinsic property of a moving body; that is, it is independent of any reference frame.	In each scenario, students measured the component displacements (d_x, d_y of the ladybug against the Cartesian grid, d_x of the surface against the fixed frame) and then determined the resultant displacement relative to the fixed frame.
The magnitude of the component velocities increases or decreases due to the interaction with the other component.	For every scenario students determined the displacement of the ladybug toy relative the Cartesian grid and the resultant displacement relative to the fixed frame. These values were then used to determine the ladybug toy's velocity and the resultant velocity. The ladybug toy's velocity on the was consistent throughout the lab and did not depend on the velocity of the surface against the frame of reference.
Speed is an intrinsic property of a moving body, and it is independent of any reference frame.	For each scenario students determined the component velocities and then determined the resultant velocity relative to the fixed frame.
The time required for a moving object to travel the resultant path is less than the amount of time required to travel the vertical or horizontal components.	Students only recorded a single time interval for each scenario that applied to all components of motion.

Feedback on Ladybug toy activities

These activities were used in whole or in part with 268 students enrolled in 9 sections of Regents Physics. My three course sections, totaling 93 students, completed the activities in their entirety as described here. Two colleagues taught the remaining six sections of physics students. These 175 students used the materials and worked through several of the scenarios. Students in these sections, however, were not asked to collect data or complete worksheet questions. For most of these groups the activity was presented after approximately one week of instruction focused on vectors.

The feedback from both colleagues about the manipulative portion of the lab was positive. Students expressed that the opportunity to complete a range of scenarios, with variations in the direction of the components, helped them understand the meaning of the arrows in vector diagrams of tip-to-tail addition. The teachers stated that students benefited from observing the concurrent perpendicular motions of the ladybug toy and the paper. In their view, this visual established a model for the concept of independent horizontal and vertical components in projectile motion.

Several weeks after the completion of these activities, the nine aforementioned sections of physics in our high school were administered the *Common Assessment of Motion* which consisted of twenty questions taken from recent Regent Physics Exams that related to kinematics. Eight of the twenty questions related to terminology or skills which were reinforced through the ladybug toy activities. Each teacher completed an analysis of student responses for each of the twenty questions and the percentages of incorrect student responses were compiled for all three teachers. The results were used as a basis for comparison and discussion of teaching practice for the topics covered on the assessment. For six of the eight items response rates were noticeably better for students who completed the entire activity. For the remainder of the assessment, scores were within two to five percent for all groups. Appendix C shows the eight vector related items on the test and the percentage of incorrect responses for each group described here.

Conclusions:

The background literature on how students learn vectors and student level of understanding conclude that instructors at the college and high school level do not realize how much difficulty students have with learning vectors (Shafer and McDermott, 2005; Knight, 1995; Nguyen and Meltzer, 2002; Arons, 1997).

In the past, I taught vectors as lead in to either projectile motion or superposition of forces and used the force table as a reinforcing activity. I have developed a new appreciation for the level of difficulty students must experienced with the traditional use of the force table as an introduction to vectors. Force tables do not encourage students to consider the frame of reference or develop an understanding of vector characteristics.

The activities described here developed a referential base of shared experience for the class, reinforced the vocabulary necessary for class discussion and allowed students to discover the essential characteristics of vector quantities. These activities required students to work through the vector operations necessary for subsequent learning in Regents Physics. By focusing attention on vector vocabulary and operations to a unit on non-accelerated motion, students were able to learn vectors in context without being overwhelmed by the difficulty of the underlying content. While students did ask pertinent questions about the concepts of displacement and velocity, they were able to understand these ideas with minimal help and devoted most of their learning to understanding vector skills. Vectors are essential for learning physics, the activities described here are a good investment of time for building a physics student's toolkit.

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Never Fall™ Wind-up toy pricing retrieved from http://www.officeplayground.com/Wind-Up-Toys-C35.aspx September 2011.

Appendix A:

Activity One - Student Worksheet: Ladybug Transit

Procedure: Trace the path of the wind-up toy as it moves around the board.

A. Assign directions on the board representing the directions of +x, -x, +y and -y

B. Fully wind the ladybug toy and place at a location near the center of the dry erase Cartesian grid.

C. Trace the motion with a dry erase marker. Use an arrow to indicate the direction of the ladybug toy. Each line is a vector. [Optional: Copy the motion of the toy onto a piece of graph paper indicating the original scale centimeters (ie, 1 block equals)]

D. A trial with a minimum of four vector arrows is considered a successful path. Each label should be labeled with a capital letter to match the data table below.

E. Determine the horizontal and vertical component of each vector by counting Cartesian grid blocks. Using a different color marker draw in horizontal and vertical vectors with appropriate arrows to indicate direction. Record the length of these vectors in the data table below. Note the sign of each motion according to Cartesian grid set-up in Step A.

	Components		**Resultant displacement (d)**			
	Horizontal X (d_x) (Block Units)	**Vertical Y (d_y)** (Block units)	**Magnitude (block units)**	**Magnitude (centimeters)**	**Reference Angle**	**Angle (given Standard Position)**
A						
B						
C						
D						
E						
R	∑	∑				

F. Determine the total horizontal and vertical components by finding the sum of each column.

G. Draw a line from the start point to the end point of the ladybug motion with an arrow pointing toward the end point. This line is called the **resultant**.

H. Determine the horizontal (d_x) and vertical (d_y) components of the resultant displacement by counting on the blocks. d_x = d_y =

I. Complete the data table by determining the magnitude and direction of each vector in the ladybug toy's path from the values of the horizontal and vertical components.

The magnitude reported in block units must be converted to centimeters. Reference angles (ie, 0-90 degrees) must be converted to standard position (0-360 degrees) based on horizontal components.

J. Write each of the vectors A-E onto individual index cards using the magnitude in centimeters and angle in standard position. Mix up order of the cards and exchange with another group. On a piece of graph paper, reproduce the path of the other team's ladybug toy and find the resultant displacement. (NOTE: You will need to determine a scale for the graph paper.) When done check your answer with the other team.

What does this tell you about the order of vectors when adding values?

Conclusion (Complete in the space below): Compare the distance of the path traveled by the ladybug toy to the displacement of the ladybug toy. Explain the difference in process for finding each value. In your comparison, explain the terms vector and scalar in terms of the concepts of distance and displacement.

Appendix B:

Activity Two - Student Worksheet: Ladybug on a conveyor belt

Purpose: To investigate and represent the motion of an object experiencing two simultaneous (concurrent) velocities. Vector addition and vector resolution will be used to analyze the motion of a wind-up ladybug toy.

Materials:

Wind-up ladybug toy
3 Meter sticks
Dry erase grid (Cartesian grid)
Large dry erase board
Stop watch

Procedure:

A. Motion of the toy is due to wind-up device. Before all experiments the toy must be fully wound.
B. The paper moves along the horizontal x axis by pulling it at roughly constant speed
C. The relative motion of the objects is found by moving both the paper and the toy between stationary meter sticks

Prior to data collection, students should practice coordinating materials and establish a realistic division of labor. See your instructor if you need assistance with this step

Data and Calculations

1. Determine the average speed of the ladybug as it moves toward the right across a stationary Cartesian grid. (Take the average of 3 trials).

Toy Data		
Distance (cm)	Time (s)	Speed (cm/s)
	AVERAGE	

2. Determine the resultant velocity of the toy and the paper if both have a rightward velocity. Attempt to move the paper at a speed similar to the toy. Fill in measurements on the table below

Time of Interval (s)	Displacement of paper relative to meter stick (cm)	Displacement of ladybug relative to the grid (cm)	Displacement of the ladybug relative to the meter sticks (resultant) (cm)	Velocity of the paper (cm/s)	Velocity of the ladybug on the paper (cm/s)	Resultant velocity of the ladybug (cm/s)

a) Show work used to determine the velocity values for the chart above

b) Construct a vector diagram that shows how the **displacement** of the ladybug toy and the displacement of the paper add to the resultant displacement of the ladybug toy. (Label each vector with a magnitude including units. Does not have to be drawn to scale but should show relative size.)

c) Construct a vector diagram that shows how the **velocity** of the ladybug toy and the velocity of the paper add to the resultant velocity of the toy when measured against the stationary dry erase board and meter sticks. (Label each vector with a magnitude including units. Does not have to be drawn to scale but should show relative size.)

d) Since the paper and the ladybug are both moving in the same direction, how would we define the angle between their motions?

3. Move the paper to the left at a similar constant speed to that of the rightward moving Ladybug for several seconds. As a group record the following. Note the direction of motion with a positive or negative sign.

Time of experiment (s)	Displacement of paper relative to meter stick (cm)	Displacement of ladybug relative to the grid (cm)	Displacement of the ladybug relative to the meter sticks (resultant) (cm)	Velocity of the paper (cm/s)	Velocity of the ladybug on the paper (cm/s)	Resultant velocity of the ladybug (cm/s)

a) Construct a labeled **displacement** diagram

b) Construct a labeled **velocity** diagram

c) What is the **difference** in direction (angle) between the papers velocity and the ladybugs velocity?

4. Vector Rule: The maximum resultant occurs when the vectors are arranged at an angle of ____________ or similar direction. The minimum resultant occurs when the vectors are arranged at an angle of ______________ or opposite direction. How could you get a different resultant without changing the magnitude (size) of the component velocities?

5. With the toy starting at the bottom left corner of the paper and pointed upward, pull the paper to the right at a pace close to that of the toy. Let it run until the toy reaches a point at the top. On the dry erase board, draw a line that connects the start and end point (ie, the resultant displacement) Fill in data on the chart below

Time of experiment (s)	Horizontal Displacement of paper relative to meter stick (cm)	Vertical Displacement of ladybug relative to the grid (cm)	Displacement of the ladybug relative to the meter sticks (resultant) (cm)	Velocity of the paper (cm/s)	Velocity of the ladybug on the paper (cm/s)	Resultant velocity of the ladybug (cm/s)
			Magnitude: Angle:			Magnitude: Angle:

Construct displacement and velocity vector diagrams showing components and resultants.

6. Predict what would happen to the angle of motion if you pull the paper at twice the speed to the right while the ladybug moves upward. Roughly draw the vectors (Hint: think of the paper as the x vector and the toy as the y vector)

7. Complete the scenario presented in the previous question. (ie Ladybug upward and paper 2X velocity right)

Time of experiment (s)	Horizontal Displacement of paper relative to meter stick (cm)	Vertical Displacement of ladybug relative to the grid (cm)	Displacement of the ladybug relative to the meter sticks (resultant) (cm)	Velocity of the paper (cm/s)	Velocity of the ladybug on the paper (cm/s)	Resultant velocity of the ladybug (cm/s)
			Magnitude: Angle:			Magnitude: Angle:

a. Draw a labeled vector diagram of the resultant displacement and horizontal and vertical components.
b. Determine the resultant velocity (magnitude and direction). Show work

8. Predict what would happen to the angle of motion if the velocity of the paper were moving to the right at a velocity about half that of the ladybug. Draw the predicted vector diagram. Complete this scenario with the materials. Does the actual motion match the prediction?

9. Draw the observed resultant path (and components) for the ladybug moving upward and the paper moving to the left (use similar speeds for both the paper and the ladybug). Label the components with the appropriate sign (+ or -).

10. Draw the observed resultant path (and components) for the ladybug starting from the top of the paper and moving downward while the paper is moving rightward. (use similar speeds for the paper and the ladybug)

11. Draw the observed resultant path (and components) for the ladybug starting from the top of the paper and moving downward while the paper is moving leftward at about twice the speed of the ladybug.

12. a) Determine the horizontal component of the motion of the paper if the ladybug travelled with a resultant speed of 14.2 cm/s and a vertical speed of 8 cm/s. Show your work including units.

    Hint: V= V_x = V_y =

b) What was the angle of the resultant for the previous problem if the ladybug was moving upward and the paper was moving left? (show work with equation and substitution)

c) What will be the resultant displacement of the ladybug after 4 seconds? Show work (include magnitude and directions)

13. When an object is launched at an angle, the initial or start velocity can be broken down into two components, velocity directed horizontally and velocity directed vertically.

What is the launch velocity of an object with a horizontal component of 40 m/s and a vertical component of 30 m/s?

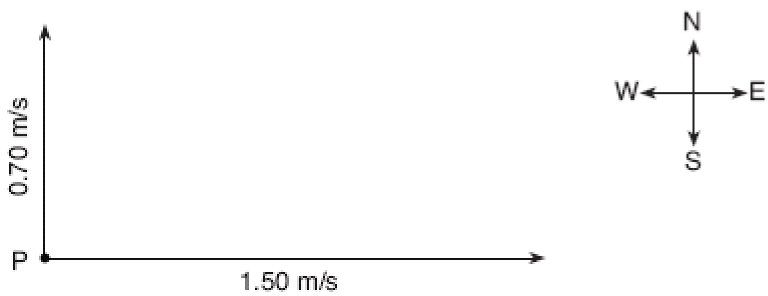
14. What are the initial horizontal and vertical components of an object's velocity if it is launched at 35 m/s at an angle of 60 degrees?

Conclusions: (to be completed on a separate paper and attached)

Explain how the addition of vector quantities is different from addition of scalar quantities. Differentiate between the terms speed and velocity. Explain why vectors are an important concept in motion and how they can be useful in other aspects of physics (Think about real life scenarios in which this applies). Summarize the methods of combining horizontal motion and vertical motion to determine a resultant and the methods for finding the horizontal and vertical components of a resultant vector. Explain how the angle between the vectors influences the magnitude of the resultant vector. Be sure to define terms used in your explanations.

APPENDIX C: Vector Problems that appeared on the *Common Assessment of Motion*

The following past Regents Physics Exam questions related to vectors appeared on an assessment given to 268 students enrolled in Regents Physics at our high school. They were part of a larger unit exam on motion. Each teacher administered the test and reported results of item analysis for the purpose of discussion within our Professional Learning Community.

	Students completing entire activity	Students completing hands-on demo only
A model airplane heads due east at 1.50 meters per second, while the wind blows due north at 0.70 meter per second. The scaled diagram below represents these vector quantities.  		
June 2011, Item # 67 On the diagram *above*, use a protractor and a ruler to construct a vector to represent the resultant velocity of the airplane. Label the vector *R*. [1]	4%	13%
June 2011, Item #68 Determine the magnitude of the resultant velocity. [1]	0.8%	7 %
June 2011, Item #69 Determine the angle between north and the resultant velocity. [1]	8%	13%
June 2010, Item #2 A motorboat, which has a speed of 5.0 meters per second in still water, is headed east as it crosses a river flowing south at 3.3 meters per second. What is the magnitude of the boat's Resultant velocity with respect to the starting point? (1) 3.3 m/s (3) 6.0 m/s (2) 5.0 m/s (4) 8.3 m/s	6%	9%

June 2008, Item #1 The speedometer in a car does *not* measure the car's velocity because velocity is a (1) vector quantity and has a direction associated with it (2) vector quantity and does not have a direction associated with it (3) scalar quantity and has a direction associated with it (4) scalar quantity and does not have a direction associated with it	3%	5%
June 2008, Item #11 An airplane flies with a velocity of 750.kilometers per hour, 30.0° south of east. What is the magnitude of the eastward component of the plane's velocity? (1) 866 km/h (3) 433 km/h (2) 650. km/h (4) 375 km/h	4%	17%
June 2008, Item #62 A kicked soccer ball has an initial velocity of 25 meters per second at an angle of 40.° above the horizontal, level ground. [Neglect friction.] Calculate the magnitude of the vertical component of the ball's initial velocity. [Show all work, including the equation and substitution with units. (Note: This question is analyzed only in terms of points lost due to incorrect application of vectors. Points lost due to missing equation or units were not included)	4%	15%
January 2008, Item #38 Two forces act concurrently on an object. Their resultant force has the largest magnitude when the angle between the forces is (1) 0° (3) 90° (2) 30° (4) 180°	7%	22%