A Hands-on Introduction to Displacement / Velocity Vectors and Frame of Reference through the Use of an Inexpensive Toy

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Abstract: This paper presents a set of hands-on activities used by the author with 93 students as an introduction to vector terminology and those operations common in the New York State Regents Physics curriculum (NYSED, 2008) through a focus on displacement and velocity vectors. Through guided activity worksheets (Appendices A and B) and the use of inexpensive equipment, students were able to visualize the tip-to-tail method of vector addition, determine the horizontal and vertical components of vectors and observe the combination of two concurrent parallel or perpendicular vectors. Students observed the motion of a wind-up toy on a moving grid within a static frame to establish the concept of frame of reference for relative motion. The terminology and level of difficulty were focused toward a high school Regents class.

Within the physics curriculum, vector operations are taught using both the Cartesian coordinate system (x,y) for the horizontal and vertical components and the polar coordinate system (R, θ) for the resultant vectors (Hoffmann, 1975). The origin of the polar coordinate axis is aligned with the positive "x" plane of the Cartesian coordinate system. The "R" serves as a symbol for any vector quantity, but displacement and velocity are substituted by students as appropriate. Cartesian and polar coordinate systems are not terms familiar to students, nor are they used in the Regents Physics curriculum. Therefore, the terms used here for Cartesian coordinate system values will be "horizontal and vertical components" which refer in equations to R_x and R_y respectively. Values reported in the polar coordinate system will be referred to by magnitude (R) and direction (θ) given in standard position or reference angle form as appropriate. To successfully complete these activities, students must be able to translate between these coordinates systems by applying the following transformation equations:

$R_x = R \cos \theta$ $R_y = R \sin \theta$ $\theta = \tan^{-1}(R_y/R_x)$ $R^2 = R_x^2 + R_y^2$

An understanding of methods used to express angles is required for reporting the direction of the resultants. Standard position refers to angles measured from the positive x-axis to the terminal side with respect to a 360 degree counterclockwise rotation (Ryan, 1993). For each angle of standard position students must be able to identify the reference angle and assign the appropriate quadrant. The reference angle is the acute angle formed by the terminal side of the given angle and the x-axis. For reference angles that do not fall in the first quadrant, students must be able to convert to standard position. Students must understand that the axes of the Cartesian coordinate system align with the quadrantal angles of 0^0, 90^0, 180^0, 270^0 and 360^0. For example, a polar coordinate vector of magnitude R at an angle of 180^0, would be written as R_x = -R, $R_y = 0$.

The table below lists the performance indicators for the NYS Regents math courses that cover content related to the required prior knowledge discussed here.

Table 2: *Integrated Algebra* (A.A.) and *Algebra 2 and Trigonometry* (A2.A) Performance Indicators

A.A.42	Find the sine, cosine, and tangent ratios of an angle of a right triangle, given the lengths of the sides.
A.A.43	Determine the measure of an angle of a right triangle, given the length of any two sides of the triangle.
A.A.44	Find the measure of a side of a right triangle, given an acute angle and the length of another side.
A.A.45	Determine the measure of a third side of a right triangle using the Pythagorean theorem, given the lengths of any two sides.
A2.A.57	Sketch and use the reference angle for angles in standard position.

(NYSED, 2005) Full text available at http://www.p12.nysed.gov/ciai/mst/math/standards/core.html

Nearly all students enrolled in Regents Physics within our school have completed *Integrated Algebra*. Therefore, a basic understanding of trigonometric functions was assumed. Approximately eighty percent of my students are concurrently enrolled in *Algebra 2 and Trigonometry*. The remaining twenty percent of students are evenly split between *Geometry* and a higher level course. Given the composition of my classes it was necessary to provide students with some formal instruction on the transformation equations and standard position angles prior to these activities. The simulation, *Vector Addition,* available through PhET, was used to reinforce the connection between the two coordinate systems and allowed students to practice with the transformation equations (http://phet.colorado.edu/en/simulation/vector-addition).

Activities

Ninety-three students, enrolled in my three sections of 31 students each, completed both activities including worksheets. *Activity One* required roughly 80 minutes for introduction, student work and discussion. *Activity Two* required roughly 90 minutes for introduction, student activities and discussion. The completion of the follow-up questions required additional class time or were assigned as homework. Both of these activities took place within a science classroom setting as they require space and a flat desk or table.

Each activity utilized a guided worksheet but required active manipulation of materials. Students worked in small groups to problem solve throughout the guided activities. The guided format was necessary due to the fact that this was an initial introduction to vectors. Students would have experienced great difficulty creating the scenarios for themselves.

Equipment:

Never Fall™ wind-up toys were used because they met the requirements of both activities. In *Activity One* it was necessary for the toy to pivot when it reached the edge of a surface. In *Activity Two,* the toy required a low center of gravity to prevent tipping when the surface grid was moved and needed to maintain a constant velocity for approximately six seconds. These toys come in several varieties and ladybug themes were selected for this class to add some levity for the adolescent audience. I also purchased and experimented with the bulldozer variety but rejected these due to a lower speed. *Never Fall*™ toys are widely available online and in toy stores for a cost of approximately $3-4 each. It is advisable to purchase extra toys to allow for malfunctions or breakage. The *Never Fall*™ wind-up toy will hereafter be referred to as the "ladybug toy."

The surface for both *Activity One* and *Activity Two* was a dry erase poster board with grid lines. These are available at teacher supply stores. For *Activity One* the grids were attached to a piece of foam board with double sided tape to provide an edge (see Figure 1a). For *Activity Two,* the movable surface should be 50-60 cm in length and a minimum of 25-35 cm in width and have a dry erase finish. A grid is helpful to verify straight line motion of the ladybug toy. An additional large dry erase board, three meter sticks and a stop watch are also required (see Figure

Several weeks after the completion of these activities, the nine aforementioned sections of physics in our high school were administered the *Common Assessment of Motion* which consisted of twenty questions taken from recent Regent Physics Exams that related to kinematics. Eight of the twenty questions related to terminology or skills which were reinforced through the ladybug toy activities. Each teacher completed an analysis of student responses for each of the twenty questions and the percentages of incorrect student responses were compiled for all three teachers. The results were used as a basis for comparison and discussion of teaching practice for the topics covered on the assessment. For six of the eight items response rates were noticeably better for students who completed the entire activity. For the remainder of the assessment, scores were within two to five percent for all groups. Appendix C shows the eight vector related items on the test and the percentage of incorrect responses for each group described here.

Conclusions:

The background literature on how students learn vectors and student level of understanding conclude that instructors at the college and high school level do not realize how much difficulty students have with learning vectors (Shafer and McDermott, 2005; Knight, 1995; Nguyen and Meltzer, 2002; Arons, 1997).

In the past, I taught vectors as lead in to either projectile motion or superposition of forces and used the force table as a reinforcing activity. I have developed a new appreciation for the level of difficulty students must experienced with the traditional use of the force table as an introduction to vectors. Force tables do not encourage students to consider the frame of reference or develop an understanding of vector characteristics.

The activities described here developed a referential base of shared experience for the class, reinforced the vocabulary necessary for class discussion and allowed students to discover the essential characteristics of vector quantities. These activities required students to work through the vector operations necessary for subsequent learning in Regents Physics. By focusing attention on vector vocabulary and operations to a unit on non-accelerated motion, students were able to learn vectors in context without being overwhelmed by the difficulty of the underlying content. While students did ask pertinent questions about the concepts of displacement and velocity, they were able to understand these ideas with minimal help and devoted most of their learning to understanding vector skills. Vectors are essential for learning physics, the activities described here are a good investment of time for building a physics student's toolkit.

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Never Fall™ Wind-up toy pricing retrieved from http://www.officeplayground.com/Wind-Up-Toys-C35.aspx September 2011.

Appendix A:

Activity One - Student Worksheet: Ladybug Transit

Procedure: Trace the path of the wind-up toy as it moves around the board.

A. Assign directions on the board representing the directions of +x, -x, +y and -y

B. Fully wind the ladybug toy and place at a location near the center of the dry erase grid.

C. Trace the motion with a dry erase marker. Use an arrow to indicate the direction of the ladybug toy. Each line is a vector. [Optional: Copy the motion of the toy onto a piece of graph paper indicating scale of original grid in centimeters (ie, 1 block equals)]

D. A trial with a minimum of four vector arrows is considered a successful path. Each label should be labeled with a capital letter to match the data table below.

E. Determine the horizontal and vertical component of each vector by counting grid blocks. Using a different color marker draw in horizontal and vertical vectors with appropriate arrows to indicate direction. Record the length of these vectors in the data table below. Note the sign of each motion according to grid set-up in Step A.

	Components		Resultant displacement (d)			
	Horizontal X (d_x) (Block Units)	Vertical Y (d_y) (Block units)	Magnitude (block units)	Magnitude (centimeters)	Reference Angle	Angle (given Standard Position)
A						
B						
C						
D						
E						
R	Σ	Σ				

F. Determine the total horizontal and vertical components by finding the sum of each column.