

This analysis was suggested by Dr. Bruce Sherwood of NCSU. Mistakes are my own:

Case 1: A small, rectangular piece of Al food wrap foil 2 cm wide in a microwave oven electric field of $2 \times 10^3 \text{ V/m}$:

Consider foil of dimension $L \gg w \gg t$, subjected to an applied electric field E_{micro} in the plane of the foil and along the width w . Because the foil is highly conductive, polarization charges will move to create two line charges $+q$ and $-q$ of length L at opposite sides of the width dimension. The dipole-like field from these two line charges when superimposed upon the original external electric field will produce an electric field of zero within the conducting foil. This situation is far from static equilibrium, but the net internal field will be practically zero due to the very high conductivity of the metal foil.

Let k be the Coulomb constant. In the center of the foil, each line charge contributes a field of

$$E_{lc} = k \frac{2q/L}{w/2} = k \frac{4q}{Lw}.$$

For the net field to vanish we must have $2E_{lc} = E_{micro}$, so

$$E_{micro} = k \frac{8q}{Lw}, \text{ or } k \frac{2q}{L} = E_{micro} \frac{w}{4}$$

The field in the air a distance r just outside the surface of the foil above a line charge q is:

$$E_{air} = k \frac{2q}{L} \frac{1}{r} = E_{micro} \frac{w}{4} \frac{1}{r} = \frac{1}{4} \frac{w}{r} E_{micro}$$

So there is a big multiplier of w/r making the field in air near the foil much stronger than the original microwave field. Assuming our sample piece of Al foil is 1 cm wide, and commercial food wrap foil is typically about 10 microns or $1 \times 10^{-5} \text{ m}$ in thickness with a cleanly rounded (semicircular) edge, so

$$r = \frac{t}{2}, \text{ then}$$

$$E_{air} = \frac{1}{4} \frac{w}{r} E_{micro} = \frac{1}{4} \frac{2 \times 10^{-2} \text{ m}}{(1 \times 10^{-5} \text{ m})/2} 2 \times 10^3 \text{ V/m} = 2 \times 10^6 \text{ V/m}$$

Which is just slightly lower than the breakdown field strength for air, $3 \times 10^6 \text{ V/m}$. Any sharp point, torn or nasty edge on the foil with a smaller curvature than $r = \frac{t}{2}$ will increase the electric field strength such that it exceeds the breakdown strength.

Case 2: Any charged conductive object with a sharp point will have highest near-surface field strengths near the sharp point.

Model our object as two spheres of radius r and R , far apart and connected by a long wire. An approximate analysis sets

$$V_{smpsh} = V_{lrgsph} \text{ so that } k \frac{q}{r} = k \frac{Q}{R} \text{ and } \frac{q}{r} = \frac{Q}{R},$$

so that the charge on the smaller sphere is

$$q = Q \frac{r}{R},$$

and the field at the surface of the small sphere is

$$E_{smpsh} = \frac{kq}{r^2} = \frac{k}{r^2} \left[Q \frac{r}{R} \right] = \frac{kQ}{rR}.$$

The field at the surface of the larger sphere is

$$E_{lrgsph} = \frac{kQ}{R^2} = \left[\frac{r}{R} \right] \frac{kQ}{rR} = \left[\frac{r}{R} \right] E_{smpsh}$$

So the field at the surface of a small sphere is a factor of R/r greater than the field at the surface of a larger connected sphere. Sharp points produce very high electric fields. Note this analysis also results in a magnification effect near the surface of an object proportional to the ratio of the largest linear dimension and the smallest.

For a very crude back-of-the-envelope approximation of the case of a CD in a microwave oven R could be as large as 6 cm (outside CD radius) and r could be as small as 2 microns (roughly the CD track spacing), hence

$$E_{track} = \left[\frac{R}{r} \right] E_{CDradius} = \left[\frac{6 \times 10^{-2} m}{2 \times 10^{-6} m} \right] \times 10^3 V/m = 6 \times 10^7 V/m$$

which significantly exceeds the dielectric breakdown strength of air. Note this analysis naively equates 2D and 3D cases, approximates measures and ignores the fact that CD foil is within a plastic cover.

Readers are also directed to the discussion of the mechanism for sparks in air by Chabay & Sherwood:

Matter & Interactions II: Electric & Magnetic Interactions, Ruth Chabay & Bruce Sherwood, Wiley 2002, pp. 484-494. The section is titled "Case Study: Sparks in air".

