Forces on the Jumping Ring

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Levitation of a metal ring by an electromagnet is a fascinating demonstration, and with readily available equipment students can make quantitative investigations of this phenomenon. With modern sensors, electronics, and computers, students can measure the factors that determine the force on a conducting ring in an alternating magnetic field.

The basic apparatus required is a solenoid with a long soft-iron core, and an aluminum or copper ring. This is the familiar Elihu Thomson "jumping ring" electromagnetism apparatus.[1] If your school does not have one, or the thousand dollars to buy one, a satisfactory solenoid can be made from a spool of heavy-gauge magnet wire, and an iron core made from a ferrite rod or a bundle of iron wire. A section of copper or aluminum pipe can be used for the ring.[2]

Theory

We will sketch the theory that is worked out by Saslow.[3] The alternating current in the solenoid is

$$I_s = I_0 \sin \omega t \tag{1}$$

where $\omega = 2\pi f$. The system produces an axial magnetic field with a radial component,

$$B_{rad} = \mu_0 n I_s K$$

$$(n = \text{turns/length of the solenoid}) \tag{2}$$

K is a geometrical constant and a measure of the flaring out of the magnetic field of the solenoid. The force on a ring of radius a along the axis is:

$$F = 2\pi a I_r B_{rad} = 2\pi a \mu_0 n K I_r I_s$$

$$(I_r = \text{current in the ring}) \tag{3}$$

By Faraday's Law, the axial component of the alternating magnetic field that passes through the ring induces a current. The ring is an LR circuit with inductance L and resistance R. It can be shown that:

$$I_r = [M I_0 \omega/(R^2 + \omega^2 L^2)](R \cos \omega t + \omega L \sin \omega t) \tag{4}$$

M is the mutual inductance. Combining Eqs. (3) and (4),

$$F = (2\pi a \mu_0 n K)[M \omega I_0^2/(R^2 + \omega^2 L^2)](R \sin \omega t \cos \omega t + \omega L \sin^2 \omega t) \tag{5}$$

The $\sin \omega t \cos \omega t$ term is equal to ½ $\sin 2\omega t$, and represents a component of the force that oscillates with a frequency of 2ω. The $\sin^2 \omega t$ term, equal to ½ $(1 - \cos 2\omega t)$, is equal to ½ when averaged over a cycle.

If the ring is kept fixed in place, the average force over a cycle of the alternating current, is:

$$\hat{F} = (\mu_0 n K M \pi a)\, \omega^2 L I_0^2/(R^2 + \omega^2 L^2) \tag{6}$$

From this we see that the force on the ring varies as the square of the current in the solenoid. The force also varies with the frequency of the ac. Other derivations can be found in any of several references.[4–6]

A qualitative way of thinking about the jumping ring is to consider that if the ring were a purely resistive circuit, the force would alternately attract and repel, and the average force would be zero. But since the ring has inductance, the induced current lags the emf by a phase angle (less than 90°) and creates an average force of

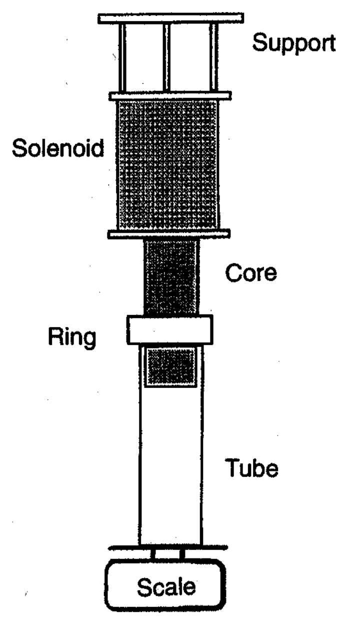

Fig. 1. Diagram of experimental apparatus.

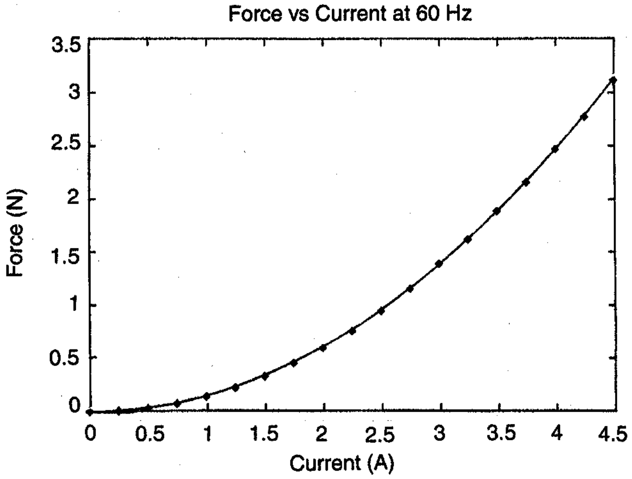

Fig. 2. Force on the ring measured for various values of the current in the solenoid. Curve is the function $y = 0.156\ x^2$. Error bars are the size of the tics.

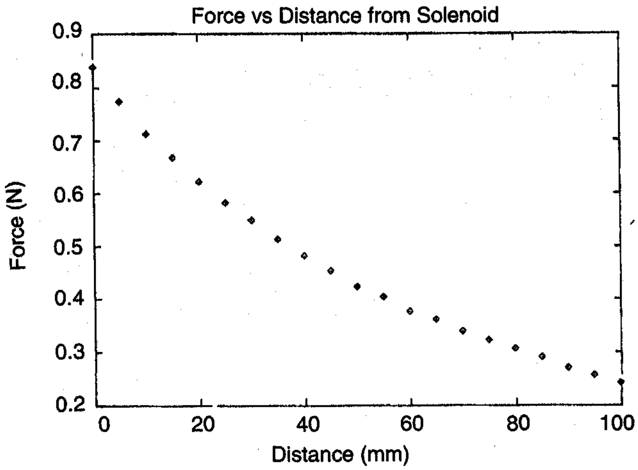

Fig. 4. Force on the ring measured at different distances between the ring and the end of the solenoid. The core extended 0.12 m out from the solenoid. Error bars are the size of the tics.

repulsion. This approach is shown in detail in a *TPT* note by A.R. Quinton.[7]

Experiments

For the following experiments, the solenoid is supported "upside down," with the ring below it (see Fig. 1). The ring is supported by a plastic pipe or cardboard tube resting on a balance on the floor. The tube must be long enough (about a meter) so that the balance is not affected by the magnetic fields.

A word of caution—do not operate the apparatus continuously with large currents. In essence, it is a step-down transformer, and with large currents the ring can easily become hot enough to burn what it touches. Also, the resistance of the ring will not be the same if the temperature varies.

A. Force on the ring as a function of current

Connect the solenoid in series with an ac ammeter and a variable transformer such as a Variac. Measure the weight of the ring and the tube supporting it with no current. Then vary the current in the solenoid and measure the force pushing down on the ring. When the scale is balanced, it will return the ring to its initial position. The data shown in Fig. 2 indicate that the force is proportional to the square of the current in the solenoid.

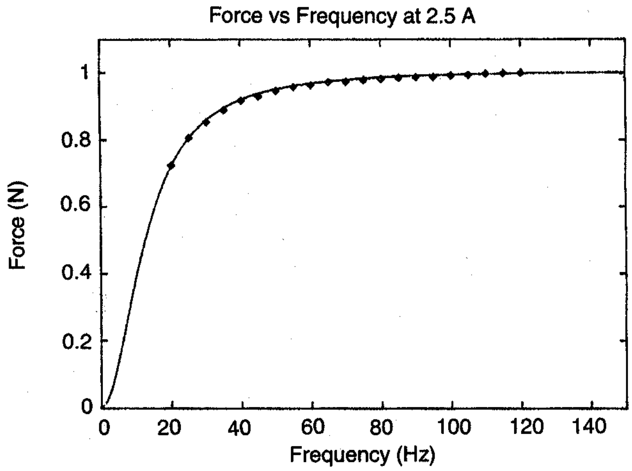

Fig. 3. Force on the ring measured at different frequencies of the alternating current in the solenoid, with $I_{rms} = 2.50$ A. Curve is the function $y = x^2/(160 + x^2)$. Error bars are about the size of the tics.

B. Force as a function of the frequency of the alternating current[8]

The jumping-ring apparatus is nearly always used with line current at a fixed frequency of 60 Hz (or 50 Hz overseas). To vary the frequency of the ac, an audio frequency amplifier with an output of several hundred watts is required. While it is not standard physics equipment, anyone involved in amplified music, in particular very loud music, has to have a powerful amplifier. From my experience, students involved in bands or car audio are usually enthusiastic in loaning an amplifier for class. Just be sure the output circuit is properly fused to avoid any damage. Your school may already have an amplifier for a public address system. A shop that deals in sales and repair of audio equipment may be a good place to find used amplifiers and technical assistance.

Connect a sine-wave generator to the input of the amplifier. To the output (usually used for the speakers), connect the solenoid in series with an ac ammeter. You will also need to be able to measure the frequency if your function generator does not indicate it. Some digital multimeters can measure frequency, as can various computer interfacing programs.

Vary the frequency of the ac, and adjust the amplitude so the current in the solenoid remains the same (the impedance of the circuit will vary with the frequency). Measure the force on the ring. Results are shown in Fig. 3.

C. Force on the ring as a function of its distance from the solenoid

Place the scale on a lab jack, or find another way to vary its height. Keeping the current in the solenoid fixed move the ring along the core and measure the force.[9] (See Fig. 4.)

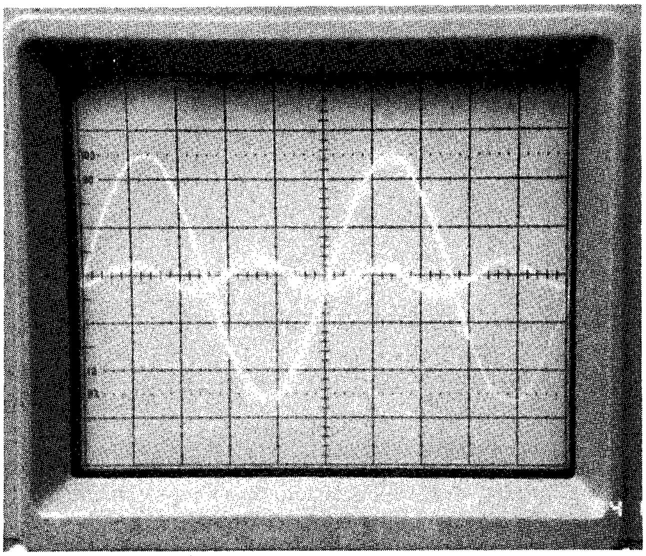

Fig. 5a. In the oscilloscope trace, the large amplitude signal is the current in the solenoid. The other signal is the alternating component of the output from the force transducer that is being pushed by the ring.

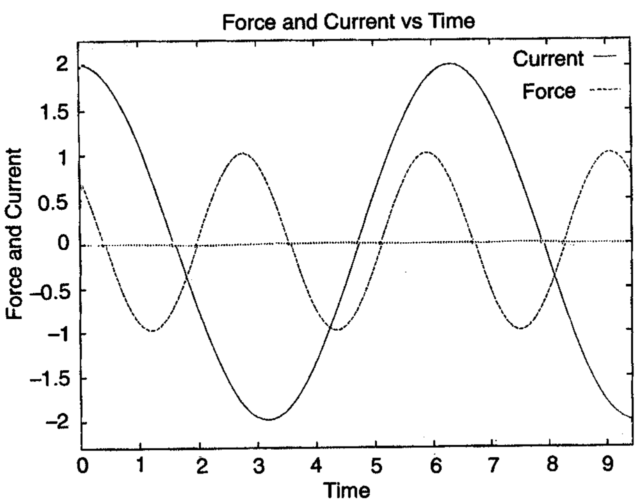

Fig. 5b. Plot of two sine functions, one at twice the frequency of the other, with a phase difference of 45° .

D. Frequency of oscillation of the force

In the previous experiments only the average force was considered. However, over the course of a cycle, the force oscillates. The force can be divided into two parts; the average force and the oscillating component. This is very similar to dividing a voltage signal into dc and ac components.

Mount the tube supporting the ring to a force transducer. Connect the output of the transducer to an oscilloscope, or to a computer interface. With a current of 5 A in the solenoid, I measured the ratio of the average force to the oscillating component to be about 30:1.

With a dual trace oscilloscope, I compared the oscillating component of the force to the current in the solenoid. As can be seen in Fig. 5, the frequency of oscillation of the force was twice the 60-H frequency of the ac. To see this, care must be taken in mounting the ring to the tube and the tube on the transducer but it is well worth the effort.

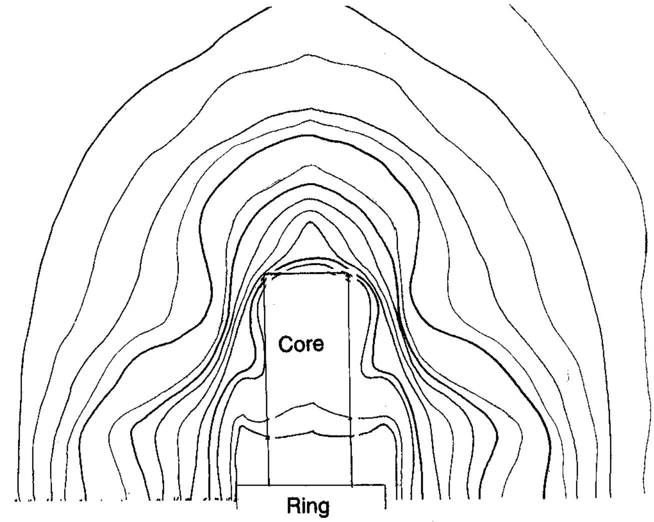

Fig. 6. Maps of (a–above) axial component and (b–below) radial component of the magnetic field about the end of the solenoid while the ring is being levitated. The difference in the magnitude of the magnetic field from one contour line to the next is 0.2 T.

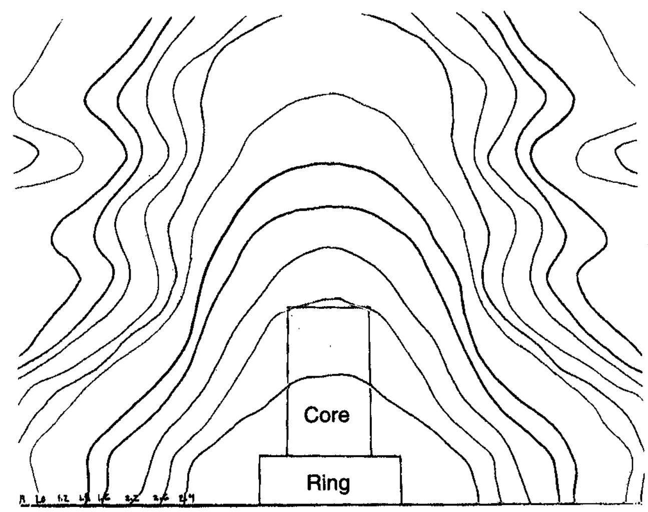

Fig. 6b.

Further Investigations

The experiments discussed so far are all "static"—the ring is kept fixed in place. There are also possible investigations of the dynamic case, allowing the ring to move. When the ring is moving, there is a viscous drag force that is a function of the ring's velocity.[9,10] What motion does the ring undergo if the amplitude or the frequency is modulated? Connecting the solenoid to amplified music turns the "jumping ring" into a "dancing ring"!

Also, the temperature of the ring can be reduced to lower the resistance, thus increasing both the current in the ring and the force. Cooling the ring in liquid nitrogen gives spectacular results.[3]

The axial and radial components of the magnetic field can be measured with low-cost Hall-effect probes and mapped (see Fig. 6).[11] Students may also wish to investigate the application of electromagnetic forces in induction motors.[2,12]

This is a wonderful time to be a teacher, and a student, of physics. With new instruments and computers, there are many phenomena to be explored in ways that would have been nearly inaccessible a few years ago. I hope that you and your students will try some of these experiments, and better yet, come up with some of your own!

Acknowledgments

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